

1. Suppose that $\sum_{k=0}^{\infty} a_k(x_0 - a)^k$ diverges for some $x_0 \in \mathbb{R}$. Show that the power series diverges for all x such that $|x - a| > |x_0 - a|$.
2. Suppose that for a power series $\sum_{k=0}^{\infty} a_k(x_0 - a)^k$ has radius of convergence R . That is:

$$R = \sup\{|x - a| : \text{the series converges for } x\}$$

which is possible ∞ .

Show:

- (a) The series converges absolutely on $(a - R, a + R)$
 - (b) The series diverges for all x for which $|x - a| > R$
3. For each of the following find the radius of convergence of the series.

(a) $\sum_{k=1}^{\infty} k(x - 2)^k$

(b) $\sum_{k=0}^{\infty} k \left(-\frac{1}{3}\right)^k (x - 2)^k$

(c) $\sum_{k=0}^{\infty} \frac{k}{3^{2k-1}} (x - 1)^{2k}$

(d) $\sum_{k=2}^{\infty} \frac{x^k}{(\ln k)^k}$