Homework Due on January 29, 2015

1. Give the definition of a set $U \subseteq \mathbb{R}$ being a **open** set.

2. Prove that if $f : [a, b] \to \mathbb{R}$ is continuous then $f([a, b])$ is bounded (In fact your task is really just finding the theorems from real anaylis I that give you this, but many proofs in this class will just be applying big theorems. Remember, you don't always have to go back to the definitions to prove everything, you can use the big theorems!)

3. State the definition of an interval (not necessarily of finite length).

4. Prove if $I$ is an interval and $f : I \to \mathbb{R}$ is continuous then $f(I)$ is an interval.

5. Show if $a, b \in \mathbb{R}$ with $a < b$ and $f : [a, b] :\to \mathbb{R}$ continuous then there exists $m, M \in \mathbb{R}$ with $m \leq M$ (note $m$ could equal $M$ so the "interval" would contain one element) such that $f([a, b]) = [m, M]$.