1. In class we definied Thomae's function $h : [0,1] \to \mathbb{R}$ as:

$$h(x) = \begin{cases} 1 & \text{if } x = 0 \\ \frac{1}{n} & \text{if } x = \frac{m}{n} \text{ where } \gcd(m,n) = 1 \\ 0 & \text{if } x \in \mathbb{R} \setminus \mathbb{Q}. \end{cases}$$

   (a) Show that h is not continuous for all $a \in \mathbb{Q}$.

   (b) Show that h is continuous for all $a \in \mathbb{R} \setminus \mathbb{Q}$.

2. Suppose that $[c,d] \subseteq [a,b]$. Remember that:

$$\mathbb{1}_{[c,d]}(x) = \begin{cases} 1 & \text{if } x \in [c,d] \\ 0 & \text{if } x \notin [c,d]. \end{cases}$$

   Show that:

   (a) $\mathbb{1}_{[c,d]} \in \mathcal{R}[a,b]$ and that $\displaystyle\int_a^b \mathbb{1}_{[c,d]} = d - c$.

   (b) $\mathbb{1}_{(c,d]}, \mathbb{1}_{(c,d)}, \mathbb{1}_{[c,d)} \in \mathcal{R}[a,b]$ and that $\displaystyle\int_a^b \mathbb{1}_{(c,d]} = \int_a^b \mathbb{1}_{[c,d)} = \int_a^b \mathbb{1}_{(c,d)} = d - c$.

3. Suppose that $f \in c([a,b])$(and further assume $f \in \mathcal{R}[a,b]$ since we have not yet proved that this is implied but we will), such that $f(x) \geq 0$ for all $x \in [a,b]$ and $\displaystyle\int_a^b f = 0$. Show $f(x) = 0$ for all $x \in [a,b]$

4. Find $f \in \mathcal{R}([a,b])$, such that $f(x) \geq 0$ for all $x \in [a,b]$ and $\displaystyle\int_a^b f = 0$, but $f(c) \neq 0$ for some $c \in \mathbb{R}$.

5. Let f be the fractional part function, i.e. $f(3.324) = 0.324$ ect.

   (a) Graph f from 0 to 10

   (b) Show that $f \in \mathcal{R}[0,10]$.

   (c) Find $\displaystyle\int_0^{10} f$.