Homework Due on February 26, 2015

1. In class we showed that if $0 < b < 1$ then:

$$\ln(1+b) = \sum_{k=1}^{n} (-1)^k \frac{b^k}{k} + R_n$$

where $R_n < b^{n+1}$. Use this (and not the way you did it on the quiz) to show that $\ln(2) < 1$.

2. More fun with $\exp(x)$ and $\ln(x)$. For this problem you may use any of the results in the previous homework (whether you proved it or not).

   (a) Let $q \in \mathbb{Q}$ and $a > 0$ show that $\ln(a^q) = q\ln(a)$.

   (b) Let $q \in \mathbb{Q}$ show that $\exp(q) = e^q$.

   (c) Suppose $a > 0$ define $a^x = \exp(x\ln(a))$. We (I mean in real analysis I) have previously defined a^q when $q \in \mathbb{Q}$, show this definition is consistent with that definition.

   (d) Let $a > 0$ and define $f(x) = a^x$, show that f is differentiable on $(-\infty, \infty)$ and find $f'(x)$.

   (e) Following what we did in class with Taylor's theorem, to show for any $b \in \mathbb{R}$:

$$\exp(b) = \lim_{n\to\infty} \sum_{k=0}^{n} \frac{b^n}{n!}$$

   Hint: I assume that in Math 360 you proved that for all $x \in \mathbb{R}$, $\lim_{n\to\infty} \frac{x^n}{n!} = 0$. In any case you may use this fact.