1. Consider the interval $[0, 6]$ and the function $f(x) = x^3$. For each of the following tagged partitions find:

   i. $\left|\left|\dot{\mathcal{P}}\right|\right|$

   ii. $\mathrm{S}(f, \dot{\mathcal{P}})$

   (a) $\dot{\mathcal{P}} = ((0, 2, 6)), (0, 5))$

   (b) $\dot{\mathcal{P}} = ((0, 1, 2, 5, 6)), (0.5, 1.5, 3.5, 5.5))$

   (c) $\dot{\mathcal{P}} = ((0, 1, 2, 3, 6)), (1, 1, 3, 3))$

2. Suppose $f \in \mathcal{R}[a, b]$ and $\dot{\mathcal{P}}_n$ be any sequence of tagged partitions such that $\lim_{n\to\infty} \left|\left|\dot{\mathcal{P}}_n\right|\right| = 0$. Show that $\lim_{n\to\infty} S(f, \dot{\mathcal{P}}_n) = \displaystyle\int_a^b f$.

3. Prove by induction that for all $n \in \mathbb{N}$, $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$. (I know this is a math logic question but do it for old times' sake.)

4. Let $f_{\text{sq}}(x) = x^2$. Assume that $f_{\text{sq}} \in \mathcal{R}[0, 1]$ (we will soon show that all continuous functions are Riemann integrable). Use the previous two problems to show that $\int_0^1 f_{sq}(x) = \frac{1}{3}$.

5. Suppose $f \in b([a, b])$ and the there exists two sequences of tagged partitions such that $\lim_{n\to\infty} \left|\left|\dot{\mathcal{P}}_n\right|\right| = 0$ and $\lim_{n\to\infty} \left|\left|\dot{\mathcal{Q}}_n\right|\right| = 0$, but $\lim_{n\to\infty} S(f, \dot{\mathcal{P}}_n) \neq \lim_{n\to\infty} S(f, \dot{\mathcal{Q}}_n)$. Show $f \notin \mathcal{R}[a, b]$.

6. Consider the function:

$$f_{\text{sp}}(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

   Prove $f_{sp} \notin \mathcal{R}[0, 1]$.