

1. Suppose $f, g \in \mathcal{R}[a, b]$ show:

(a) if $k \in \mathbb{R}$ then $kf \in \mathcal{R}[a, b]$.

(b) if $f(x) \leq g(x)$ for all $x \in [a, b]$ then $\int_a^b f \leq \int_a^b g$.

2. Let $f(x) = x$ and $a < b$. Show that $f \in \mathcal{R}[a, b]$ and find $\int_a^b f$.

3. Let $g(x) = 4x + 3$. Show $g \in \mathcal{R}[1, 4]$ and find $\int_1^4 g$.

4. Suppose $f \in \mathcal{R}[a, b]$ and $c \in \mathbb{R}$, we define $g : [a + c, b + c] \rightarrow \mathbb{R}$ by $g(x) = f(x - c)$. Prove that $g \in \mathcal{R}[a + c, b + c]$ and:

$$\int_{a+c}^{b+c} g = \int_a^b f$$

The function g is called to c -translate of f .