Homework Due on March 12, 2015

1. For $x, y \in \mathbb{R}$ find $C(x+y)$ and $C(-x)$ in terms of $S(x)$ and $C(x)$.

2. Let f be a periodic function with period p and domain D (i.e. p is the smallest positive number such that $f(x+p) = f(x)$ for all $x \in D$).

   (a) Show that for all $k \in \mathbb{Z}$ and $x \in D$, $f(x+kp) = f(x)$.

   (b) Suppose $a \in \mathbb{R}$ is such that $f(x+a) = f(x)$ for all $x \in D$. Show that there exists $k \in \mathbb{Z}$ such that $a = kp$.

3. Let $\cosh(x) = \dfrac{\exp(x) + \exp(-x)}{2}$ and $\sinh(x) = \dfrac{\exp(x) - \exp(-x)}{2}$.

   (a) Show that $\cosh'(x) = \sinh(x)$ and $\sinh'(x) = \cosh(x)$ with $\cosh(0) = 1$ and $\sinh(0) = 0$.

   (b) Show that $\cosh^2(x) - \sinh^2(x) = 1$ for all $x \in \mathbb{R}$.

   (c) Show that if f is twice differentiable on $\mathbb{R}$ with $f''(x) = f(x)$ then there exists α and β such that $f(x) = \alpha \cosh(x) + \beta \sinh(x)$. Also find α and β in terms of f.

   (d) Find $\sinh(x+y)$ and $\cosh(x+y)$ for $x, y \in \mathbb{R}$ in terms of $\sinh(x)$ and $\cosh(x)$.

   (e) Determine if sinh and cosh are odd, even or neither (for extra admiration use (c) rather than the definition to prove this.)