Homework Due on March 17, 2015

1. (a) Show that the only roots of the C function on $[0, 2\pi]$ are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

   (b) Show that C is periodic with period 2π.

2. Show that $3 \leq \pi \leq 2\sqrt{6 - 2\sqrt{3}}$. (Hint use $C(x)$)

3. (a) Show that S has roots when $x = k\pi$ where $k \in \mathbb{Z}$.

   (b) Show these are the only roots of S. (Hint spend some time thinking about this and use the greatest integer function).

4. Show that if $0 \leq \alpha, \beta \leq \frac{\pi}{2}$ and $\frac{\pi}{2} < \alpha + \beta \leq \pi$. Then:

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)$$
$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

5. Show for all $\alpha, \beta \in \mathbb{R}$ then:

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)$$
$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$