Homework Due on March 24, 2015

1. Remember now that sin and cos are the same as what we called S and C so you can use all the properties we showed for S and C. Define $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$.

   (a) Show $\tan(x) : (-\frac{\pi}{2}, \frac{\pi}{2}) \to \mathbb{R}$ is an increasing, continuous, bijection.

   (b) Define $\arctan(x) : \mathbb{R} \to (-\frac{\pi}{2}, \frac{\pi}{2})$ to be the functional inverse of $\tan(x)$. Show it is an increasing, continuous, bijection.

   (c) Find $\arctan'(x)$.

2. Show if $\sum_{k=1}^{\infty} a_k$ converges then $\lim_{k\to\infty} a_k = 0$.

3. Suppose for some $M > 0$, $0 \le Mb_k \le a_k$ eventually. Show that if $\sum b_k$ diverges, then $\sum a_k$ diverges.

4. Suppose that $\left|\dfrac{a_{k+1}}{a_k}\right| > 1$ eventually. Show that $\sum a_k$ diverges.