Homework Due on March 26, 2015

1. Let a_n defined as:
$$a_n = \begin{cases} \frac{1}{2^{(n+1)/2}} & \text{if } n \text{ is odd} \\ \frac{1}{3^{n/2}} & \text{if } n \text{ is even} \end{cases}$$
Show that $\sum_{k=0}^{\infty} a_k$ converges.

2. Suppose that $\sum_{k=0}^{\infty} a_k(x_0 - a)^k$ diverges for some $x_0 \in \mathbb{R}$. Show that the power series diverges for all x such that $|x - a| > |x_0 - a|$.

3. Suppose that for a power series $\sum_{k=0}^{\infty} a_k(x_0 - a)^k$ has radius of convergence R. That is:
$$R = \sup\{|x - a| : \text{ the series converges for } x\}$$
which is possible ∞.
Show:
   (a) The series converges absolutely on $(a - R, a + R)$
   (b) The series diverges for all x for which $|x - a| > R$

4. For each of the following find the radius of convergence of the series.
   (a) $\sum_{k=1}^{\infty} k(x - 2)^k$
   (b) $\sum_{k=0}^{\infty} k\left(-\frac{1}{3}\right)^k (x - 2)^k$
   (c) $\sum_{k=0}^{\infty} \frac{k}{3^{2k-1}}(x - 1)^{2k}$
   (d) $\sum_{k=2}^{\infty} \frac{x^k}{(\ln k)^k}$