Homework Due on March 05, 2015

1. Show that if $\{f_n\}$ converges to f uniformly on $A \subseteq \mathbb{R}$ then it converges pointwise on A.
2. Show that $\{f_n\}$ converges to some f uniformly on $A \subseteq \mathbb{R}$ then $\{f_n\}$ is Cauchy with respect to the uniform norm.
3. Show that if $f(x)$ is differentiable on $\mathbb{R}$ and for all $x \in \mathbb{R}$, $f'(x) = f(x)$ and $f(0) = 1$ then $f(x) = \exp(x)$.
4. Let $0 < L < 1$ and $f_n(x) = \sum_{k=0}^{n} x^k$. Show that f_n converges uniformly to some f on $[-L, L]$.
   (a) Show that f_n converges to $\dfrac{1}{1-x}$ pointwise on $(-1, 1)$.
   (b) Show that f_n converges uniformly to some f on $[-L, L]$.
   (c) Find:
   $$\lim_{n\to\infty} \int_{-\frac{1}{3}}^{\frac{1}{2}} f_n.$$