Homework Due on April 30, 2015

1. Let $X = \{f : [1,4] \to \mathbb{R} : f \text{ is a polynomial }\}$ with the norm $||\cdot||_\infty$. Let $h \in X$ be defined by $h(x) = x^2 - 4x - 3$. Let $I \in X^*$ be defined by $I(f) = \int_1^4 2f$. For each $a \in [1,4]$ and $f \in X$ let $E_a(f) = f(a)$. For $f \in X$ let $\mathcal{F}_f \in X^{**}$ be defined by $\mathcal{F}_f(T) = T(f)$. Finally let $F : X \to X^{**}$ be defined by: $F(x) = \mathcal{F}_x$. All norms below are the norms on their respective spaces. (Hint: Don't let the notation confuse you.)

   (a) Find $||h||$.

   (b) Find $||I||$.

   (c) Show $E_a \in X^*$ for all $a \in [1,4]$.

   (d) Find $||E_a||$.

   (e) Without the Hahn-Banach Theorem find $||\mathcal{F}_h||$.

   (f) In class we showed that the Hahn-Banach Theorem implies that F is an isometric embedding of X into X^{**} (i.e. an injective norm-preserving map). Show $F(X) \neq X^{**}$(i.e. F is not onto).