Homework Due on April 7, 2015

1. Suppose $\{a_n\}$ is a sequence. Show:

   (a) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \beta < 1$ then $\sum_{k=0}^{\infty} a_k$ converges absolutely.

   (b) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \beta > 1$ then $\sum_{k=0}^{\infty} a_k$ diverges.

2. Let $a_n$ defined as:

$$a_n = \begin{cases} \frac{1}{2^{(n+1)/2}.3^{(n-1)/2}} & \text{if } n \text{ is odd} \\ \frac{1}{2^{n/2}.3^{n/2}} & \text{if } n \text{ is even} \end{cases}$$

   Show that $\sum_{k=0}^{\infty} a_k$ converges.

3. Consider the power series: $\sum_{k=1}^{\infty} kx^k$.

   (a) Find its radius of convergence

   (b) In the interior of the interval of convergence, give a closed form expression (one without summation) for what the function the series converges to.

   (c) Find the sum of $\sum_{k=1}^{\infty} \frac{k}{2^k}$.

4. Consider the power series: $\sum_{k=1}^{\infty} k^2x^k$.

   (a) Find its radius of convergence

   (b) In the interior of the interval of convergence, give a closed form expression (one without summation) for what the function the series converges to.

   (c) Find the sum of $\sum_{k=1}^{\infty} \frac{k^2}{3^k}$.

5. Let $X$ be a nonempty set and define:

$$d(x, y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y. \end{cases}$$

   Show $(X, d)$ is a metric space.