Homework Due on April 9, 2015

1. Suppose (X, d_X) and (Y, d_Y) are metric spaces. Show that $f : X \to Y$ is continuous (on X) if and only if for every open set U of Y, $f^{-1}(U)$ is open in X.

2. Show $x_n \to x$ in a metric space (X, d) if and only if for all open sets U in X containing x, there exists $N \in \mathbb{N}$ such that $x_n \in U$ for all $n \geq N$.

3. We say that $A \subseteq X$ is closed in a metric space (X, d) if whenever $\{x_n\} \subseteq A$ and $x_n \to x \in X$, then $x \in A$ (i.e. the set is closed under sequence convergence). Show that $A \subseteq X$ is closed if and only if $X \setminus A$ is open in X.

4. Suppose that $(X, ||\cdot||)$ is a normed vector space. Show that if $d : X \times X \to \mathbb{R}$ is defined by $d(x, y) = ||x - y||$ then (X, d) is a metric space.