

Math 370 Number Theory Assignment # 6

1. Fifteen pirates steal a stack of identical gold coins. When they try to divide them evenly, two coins are left over. A fight erupts and one of the pirates is killed. The remaining pirates try again to evenly distribute the coins. This time there is one coin left over. A second pirate is killed in the resulting argument. Now when the remaining pirates try to divide the coins evenly there are no coins left over.

What is the smallest number of coins that could have been in the sack?

2. Let $f : \mathbb{N} \rightarrow \mathbb{C}$ be arithmetic (and not the 0 function).
 - (a) Prove $f(1) = 1$.
 - (b) Suppose a_1, a_2, \dots, a_k are pairwise relatively prime (that is any pair are relatively prime). Use induction to show that $f(a_1 a_2 a_3 \dots a_k) = f(a_1) f(a_2) f(a_3) \dots f(a_k)$.
 - (c) Suppose $f(12) = 9, f(21) = 3, f(28) = 3$. What are the possible values for $f(84)$?
3. Suppose you know that $n = 2500969819 = pq$, where p and q are prime. Find p, q using the fact that $\phi(n) = 2500846920$.