Math 370 Number Theory
Assignment # 2

1. Suppose that $a, b \in \mathbb{Z}$ with $a \neq 0$. Show $\gcd(a,b) = \gcd(-a,b)$.

2. Use the Euclidian Algorithm and the above problem to find $\gcd(-209287, -2023)$.

3. Finish the proof of the division algorithm proving by that if $a \in \mathbb{Z}$ with $a < 0$ and $b \in \mathbb{N}$ then there exist $q, r \in \mathbb{Z}$ such that $a = q \cdot b + r$ and $0 \leq r < b$. (Hint: in class we prove that this holds when $a \geq 0$ so you can use this fact in your proof. We also proved uniqueness in all cases so you don't need to do that).

4. Let $a, b \in \mathbb{Z}$ be not both 0.
   (a) Show that the Diophantine equation $ax + by = d$ has a solution for $x, y \in \mathbb{Z}$ if and only if $g|d$ where $g = \gcd(a,b)$.
   (b) Show that a and b are relatively prime if and only if there exists $x, y \in \mathbb{Z}$ such that $ax + by = 1$.
   (c) Use the above to give a different proof of the following theorem you proved in the last homework:
   Suppose a and b are positive integers with $g = \gcd(a,b)$, prove $\gcd(a/g, b/g) = 1$.

5. Suppose that a and b are relatively prime, prove that if $m \in \mathbb{Z}$ and $a|mb$ then $a|m$. (Hint: There is a very nice and easy way to do this.)