

Math 370 Number Theory Assignment # 5

1. (a) Let $a, b \in \mathbb{Z}$ and $m \in \mathbb{N}$. Show if a and b are both relatively prime to m then ab is relatively prime to m . (Hint: You have essentially already proved this in a previous homework so it should be easy).
(b) Suppose that $b_1, b_2, \dots, b_n \in \mathbb{Z}$ are each relatively prime to $m \in \mathbb{N}$. Use induction to prove that $b = b_1 \cdot b_2 \cdot b_3 \cdot \dots \cdot b_n$ is relatively prime to m .
2. Suppose $a \in \mathbb{Z}$ and p is prime. Show that $a^p \equiv a \pmod{p}$. (Note that we are not assuming anything about a .)
3. (a) Suppose the p is prime and $a \in \mathbb{Z}$ such that $a^2 \equiv 1 \pmod{p}$. Show that either $a \equiv 1 \pmod{p}$ or $a \equiv -1 \pmod{p}$.
(b) Find $m \in \mathbb{N}$ and $a \in \mathbb{Z}$ such that $a^2 \equiv 1 \pmod{m}$ but neither $a \equiv 1 \pmod{m}$ nor $a \equiv -1 \pmod{m}$.