

## Math 370 Number Theory Assignment # 3

1. Prove that if  $a, b, c \in \mathbb{Z}$  such that  $a|b$  and  $b|c$  then  $a|c$ . (Try to do this by yourself without notes, formally and carefully)
2. Let that  $a, b \in \mathbb{N}$  be relatively prime. Show that if  $a|c$  and  $b|c$  then  $ab|c$ .
3. (a) Prove that  $2|n(n+1)$  for all  $n \in \mathbb{N}$ .  
(b) Prove by induction that  $6|n(n^2+5)$  for all  $n \in \mathbb{N}$ .  
(c) Let  $a \in \mathbb{R}$  with  $a \neq 1$ . Prove by induction that for all  $n \in \mathbb{N}$ :

$$1 + a + a^2 + \dots + a^n = \frac{1 - a^{n+1}}{1 - a}$$

4. In each case determine if the Diophantine equation has a solution. If it does find two solutions.
  - (a)  $21284x + 354756y = 68$
  - (b)  $25704x + 249288y = 25$
5. Like in chapter 7, let  $\mathbb{E} = \{\dots, -4, -2, 0, 2, 4, \dots\}$ .
  - (a) Write down the first 10 “primes” (positive elements in  $\mathbb{E}$  that are not the product of two elements in  $\mathbb{E}$ .)
  - (b) Make a conjecture about when a number is an element of  $\mathbb{E}$  is prime. (Hint: There is a very simple condition.)
  - (c) Prove this conjecture.
6. Factor the number 97881827. (Hint: all the prime factors are less than 300 so don't panic).