Math 370 Number Theory
Assignment # 7

1. (a) Suppose that $k, m, n \in \mathbb{N}$ such that $\gcd(m, n) = 1$. Let $a = \gcd(k, m)$ and $b = \gcd(k, n)$ and $k|mn$. Show that:
      i. $\gcd(a, b) = 1$.
      ii. $ab|k$.
      iii. Show that $k|ab$ and hence k=ab. (Hint: Use Bézout's theorem for both a and b).
   (b) Show if m, n are relatively prime natural numbers then $k|mn$ if and only if $k = ab$ where $a|m$ and $b|n$ and $\gcd(a, b) = 1$.

2. For $n \in \mathbb{N}$, define:
$$\sigma(n) = \sum_{d|n} d.$$
   That is $\sigma(n)$ is the sum of all divisors on n.
   (a) Find:
      i. $\sigma(15)$
      ii. $\sigma(4)$
      iii. $\sigma(60)$
   (b) Show that σ is arithmetic multiplicative. (Hint use the result from Question 1).

3. Let p be an odd prime and $a \in \mathbb{Z}$ with $p \nmid a$. Show that if $x^2 = a \pmod{p}$ has a solution, then it has exactly 2 solutions (up to congruence i.e. there are two solutions between 0 and $p - 1$). (Hint let b be a solution and then show there will be exactly 2).