Math 370 Number Theory
Assignment # 4

1. Let $m \in \mathbb{N}$. Show that $\equiv \pmod{m}$ is an equivalence relation. Hence show:
   (a) That it is reflective, that is show that for all $a \in \mathbb{Z}$, $a \equiv a \pmod{m}$.
   (b) That it is symmetric, that is show that for all $a, b \in \mathbb{Z}$ if $a \equiv b \pmod{m}$ then $b \equiv a \pmod{m}$.
   (c) That it is transitive, that is show for all $a, b, c \in \mathbb{Z}$, if $a \equiv b \pmod{m}$ and $b \equiv c \pmod{m}$ then $a \equiv c \pmod{m}$.

2. Consider the congruence equation:

$$627111x \equiv 9145 \pmod{2538239}.$$

   (a) How many non-congruent solutions does this equation have?
   (b) If there are any solutions, find the first three positive solutions.

3. For each of the following find $a^{-1} \pmod{m}$ or explain why it doesn't exist.
   (a) $a = 431262, m = 205715369$
   (b) $a = 47, m = 2160992$

4. Suppose that $a, k \in \mathbb{Z}$ and $m \in \mathbb{N}$ and $\gcd(a, m) = 1$ and $\gcd(k, m) = 1$. Show that if $ak \equiv b \pmod{m}$ then $\gcd(b, m) = 1$.