

1. Show if m, n are relatively prime then $k|mn$ if and only if $k = ab$ where $a|m$ and $b|n$.
2. Show if n_1, \dots, n_r are pairwise relatively prime then $k|n_1n_2 \dots n_r$ if and only if $k = a_1a_2 \dots a_r$ where $a_i|n_i$.
3. Suppose $n = p_1^{\alpha_1}p_2^{\alpha_2} \dots p_r^{\alpha_r}$ with $1 \leq \alpha_i$ and p_i prime, is the unique prime factorization of n . Show that $a|n$ if and only if $a = p_1^{\beta_1}p_2^{\beta_2} \dots p_r^{\beta_r}$ where $0 \leq \beta_i \leq \alpha_i$. This shows that the unique factorization of a must include only primes in the factorization of n and must have smaller (or equal) powers of each of those primes.