

Collected Problems:

1. Show that if  $p$  is an odd prime,  $p \nmid a$ ,  $1 \leq i \leq \frac{p-1}{2}$ , and  $1 \leq j \leq \frac{p-1}{2}$  then if  $ai \equiv -aj \pmod{p}$  then  $i = j$ .
2. For each of the following compute the value of the Legendre symbol using (a)Euler's criterion (b)Gauss's Lemma: (do not use gp or a similar calculator, show work.)
  - (a)  $\left(\frac{2}{29}\right)$
  - (b)  $\left(\frac{3}{11}\right)$
  - (c)  $\left(\frac{13}{17}\right)$
3. Calculate: (in each the denominator is prime)
  - (a)  $\left(\frac{50}{17827}\right)$
  - (b)  $\left(\frac{917436167}{917804249}\right)$  (Hint:use negative numbers)
4. If  $p$  is congruent to 1 modulo 5, then 5 is a quadratic residue modulo  $p$ . (Hint: Use Gauss's lemma)