

Collected Problems:

1. Calculate the following Jacobi Symbol:

(a)  $\left(\frac{5}{21}\right)$

(b)  $\left(\frac{111}{1001}\right)$

(c)  $\left(\frac{1009}{2307}\right)$

2. For which positive integers  $n$  that are relatively prime to 15 does the Jacobi symbol  $\left(\frac{15}{n}\right) = 1$ .

3. Suppose that  $n = pq$ , where  $p$  and  $q$  are odd primes. We say that the integer  $a$  is a pseudo-square modulo  $n$  if  $a$  is a quadratic nonresidue of  $n$ , but  $\left(\frac{a}{n}\right) = 1$ .

(a) Show that if  $a$  is a pseudo-square modulo  $n$ , then  $\left(\frac{a}{p}\right) = \left(\frac{a}{q}\right) = -1$ .

(b) Find all the pseudo-squares modulo 21.

4. Let  $n = 15841$ .

(a) Factor  $n$  to show that  $n$  is not prime.

(b) Show the  $n$  is a Carmichael number.

(c) Show 2 is not an Euler witness for  $n$ .

(d) Show 2 is not a Rabin-Miller witness for  $n$ .

Non-Collected Problems:

1. Calculate the following Jacobi Symbol:

(a)  $\left(\frac{2663}{3299}\right)$

(b)  $\left(\frac{10001}{20003}\right)$

2. Let  $n$  be an odd square-free number (i.e. all of its prime factors are unique). Show that there is an integer  $a$  such that  $\gcd(a, n) = 1$  and  $\left(\frac{a}{n}\right) = -1$ .