Math 370 Number Theory
Assignment # 2

1. Let $a, b \in \mathbb{Z}$ be not both 0. (Do this problem on a separate paper and don't staple it so that a student can critique it.)
   (a) Show that the Diophantine equation $ax + by = d$ has a solution for $x, y \in \mathbb{Z}$ if and only if $g|d$ where $g = \gcd(a, b)$.
   (b) Show that a and b are relatively prime if and only if there exists $x, y \in \mathbb{Z}$ such that $ax+by = 1$.
   (c) Use the above to give a different proof of the following theorem you proved in the last homework:
   Suppose a and b are positive integers with $g = \gcd(a, b)$, prove $\gcd(a/g, b/g) = 1$.

2. Suppose that a and b are relatively prime, prove that if $m \in \mathbb{Z}$ and $a|mb$ then $a|m$. (Hint: There is a very nice and easy way to do this.)

3. In each case determine if the Diophantine equation has a solution. If it does find two solutions.
   (a) $21284x + 354756y = 68$
   (b) $25704x + 249288y = 25$