

Math 370 Number Theory Assignment # 1

1. Prove that if a is a positive integer then $\gcd(a, 0) = a$ (Don't use the Euclidian Algorithm since this result is needed to prove the Euclidian Algorithm is sound).
2. Suppose d is a common divisor to a and b . Prove that $d|(ma + nb)$ for all $m, n \in \mathbb{Z}$.
3. Suppose a and b are positive integers with $g = \gcd(a, b)$, prove $\gcd(a/g, b/g) = 1$.
4. For each of the following a and b find the unique q and r such that $a = q \cdot b + r$ and $0 \leq r < b$.
 - (a) $a = 12321, b = 121$
 - (b) $a = -12321, b = 121$
 - (c) $a = 621243, b = 1211$
 - (d) $a = -621243, b = 1211$
 - (e) $a = 6660, b = 555$
 - (f) $a = -6660, b = 555$
5. Prove the division algorithm holds when $a \leq 0$. That is prove that if $a \in \mathbb{Z}$ with $a \leq 0$ and $b \in \mathbb{N}$ then there exist unique $q, r \in \mathbb{Z}$ such that $a = q \cdot b + r$ and $0 \leq r < b$.
6. Show there is no infinite and strictly decreasing sequence of non-negative integers.