Math 370 Number Theory
Assignment # 5

1. The following was used in the proof of Euler's theorem so once you proved it you will have finished the proof (350 years to late to get much credit sadly). Put this on a separate page. We are going to do a peer review again.

   (a) Let $a, b \in \mathbb{Z}$ and $m \in \mathbb{N}$. Show if a and b are both relatively prime to m then ab is relatively prime to m. (Hint: You have essentially already proved this in a previous homework so it should be easy).

   (b) Suppose that $b_1, b_2, \ldots, b_n \in \mathbb{Z}$ are each relatively prime to $m \in \mathbb{N}$. Use induction to prove that $b = b_1 \cdot b_2 \cdot b_3 \cdot \cdots \cdot b_n$ is relatively prime to m.

2. Suppose $a \in \mathbb{Z}$ and p is prime. Show that $a^p \equiv a \pmod{p}$. (Note that we are not assuming anything about a.)

3. Fifteen pirates steal a stack of identical gold coins. When they try to divide them evenly, two coins are left over. A fight erupts and one of the pirates is killed. The remaining pirates try again to evenly distribute the coins. This time there is one coin left over. A second pirate is killed in the resulting argument. Now when the remaining pirates try to divide the coins evenly there are no coins left over.

   Answer one of the following two questions:

   (a) What is the smallest number of coins that could have been in the sack?

   (b) What socio-economic factors lead to pirates to be so violent. Are outcast communities necessarily lawless? Make sure to use examples from Anton Chekhov's *Three Sisters* at least one primary source in your response.

4. Let $f : \mathbb{N} \to \mathbb{C}$ be arithmetic (and not the 0 function).

   (a) Prove $f(1) = 1$.

   (b) Suppose $a_1, a_2, \ldots, a_k$ are pairwise relatively prime (that is any pair are relatively prime). Use induction to show that $f(a_1 a_2 a_3 \ldots a_k) = f(a_1)f(a_2)f(a_3) \ldots f(a_n)$.

   (c) Suppose $f(12) = 9, f(21) = 3, f(28) = 3$. What are the possible values for $f(84)$?