

Math 370 Number Theory Assignment # 3

1. Prove that if $a, b, c \in \mathbb{Z}$ such that $a|b$ and $b|c$ then $a|c$. (This is a simple but useful result that I meant to have you do earlier but forgot and we might have even used it already).
2. Suppose that $a, b \in \mathbb{Z}$ and aren't both 0 and $g = \gcd(a, b)$. Show that if d is a common divisor of a and b if and only if $d|g$.
3. Like in chapter 7, let $\mathbb{E} = \{\dots, -4, -2, 0, 2, 4, \dots\}$.
 - (a) Write down the first 10 "primes" (positive elements in \mathbb{E} that are not the product of two elements in \mathbb{E} .)
 - (b) Make a conjecture about when a number is an element of \mathbb{E} is prime. (Hint: There is a very simple condition.)
 - (c) Prove this conjecture.
4.
 - (a) Prove that $2|n(n+1)$ for all $n \in \mathbb{N}$.
 - (b) Prove by induction that $6|n(n^2+5)$ for all $n \in \mathbb{N}$.
5. Factor the number 97881827. (Hint: all the prime factors are less than 300 so don't panic).
6. Find $\text{lcm}(47146968, 37146096)$ using the Euclidian algorithm.