

Math 370 Number Theory Assignment # 11

1. Suppose $a, b \in \mathbb{N}$ be odd and $m \in \mathbb{Z}$. Show that:

$$\left(\frac{m}{ab}\right) = \left(\frac{m}{a}\right) \left(\frac{m}{b}\right)$$

2. Suppose $n \in \mathbb{N}$ be odd. Show (Hint: Use induction on the number of primes in the factorization of n):

(a)

$$\left(\frac{-1}{n}\right) = \begin{cases} 1 & \text{if } n \equiv 1 \pmod{4} \\ -1 & \text{if } n \equiv 3 \pmod{4} \end{cases}$$

(b)

$$\left(\frac{2}{n}\right) = \begin{cases} 1 & \text{if } n \equiv 1 \text{ or } -1 \pmod{8} \\ -1 & \text{if } n \equiv 3 \text{ or } 5 \pmod{8} \end{cases}$$

3. (a) Let $n \in \mathbb{N}$ be an odd such that $5 \nmid n$. Find a formula for $\left(\frac{5}{n}\right)$ that involves what n is congruent to mod 5.

(b) If p is an odd prime. What must its final digit be for 5 to be a QR mod p ?

(c) If m is a perfect square and $5 \nmid m$ what are the possibilities for $m \pmod{5}$?

(d) If m is a perfect square and $5 \nmid m$ what are the possibilities for the last digit of m ?

(e) If m is a perfect square what are the possibilities for the last digit of m ?