Additional Group Problems Assignment 9

1. Consider the group $U(27)$.
   (a) List the elements of the group.
   (b) Show that $\langle 5 \rangle = U(27)$
   (c) Find all other generators of the group. (Hint: There is an easy way to do this and a hard way).
2. Show that if $U(n)$ is cyclic it has $\phi(\phi(n))$ generators.
3. Show that if G is a group and $H \leq G$ and $a \in G$. Then $a \in H$ if and only if $\langle a \rangle \subseteq H$.
4. You have previously shown that if G is a group and H and K are subgroups of G then $H \cap K$ is a subgroup of G. Produce an example of a group G and two subgroups H and K of G such that $H \cup K$ is not a group. (This is how the problem should have read last week. Of course the way it did read was too easy.)