Additional Problem Assignment 6

Please do each of these problems on a separate sheet of paper, but not necessarily for each part of a problem.

1. Consider the group D_4. Check the so called "Socks and Shoes" Property. But checking the following:
   (a) $(HV)^{-1} = V^{-1}H^{-1}$
   (b) $(R_{90}D)^{-1} = D^{-1}R_{90}^{-1}$
2. (a) Show that for all $n \in \mathbb{N}$, $n-1 \in U(n)$.
   (b) Show that $n-1$ is its own inverse in $U(n)$.
3. (a) Show if $M_1 = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$ and $M_2 = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$ then $\det(M_1M_2) = \det(M_1)\det(M_2)$.
   (b) Use the previous part to show that if $M \in GL(2,\mathbb{R})$ then $\det(M^{-1}) = \dfrac{1}{\det M}$.
4. Suppose $H = \{M \in GL(2,\mathbb{R}) : \det M \in \mathbb{Q}\}$.
   (a) Find an element of H that is not in $GL(2,\mathbb{Q})$ (i.e. $H \not\subseteq GL(2,\mathbb{Q})$).
   (b) Show H is a subgroup of $GL(2,\mathbb{R})$.