Additional Problems Assignment 14

1. In class we defined that for two groups $G_1 \cong G_2$ if there exists $\phi : G_1 \to G_2$ such that ϕ is a bijection and for all $a, b \in G_1$, $\phi(ab) = \phi(a)\phi(b)$. Show that $\cong$ is an equivalence class on the class of Groups. As you might remember from logic that includes showing three things:
   (a) For any group G, $G \cong G$ (reflexivity)
   (b) For any groups G_1, G_2 if $G_1 \cong G_2$ then $G_2 \cong G_1$. (Symmetry)
   (c) For any groups G_1, G_2, G_3 if $G_1 \cong G_2$ and $G_2 \cong G_3$ then $G_1 \cong G_3$. (Transitivity)