Additional Problems Assignment 21

Last time for the exam, I gave you detailed problems to do. This time I am going to make you work harder and learn how to study for an exam. Remember that these problems are for your own good and since you aren't taking this exam in groups you need to make sure you understand the concepts and how to do the problems, yourself. So make sure you get the most out of it and you know how to do the problems. You will not turn in this assignment, it is for you!

1. In each of the section this exam covers (Chapters 5-9) write down each of the important definitions and know them.
2. Give an example of each of the following or explain why no such example exists. I choose one from each section. Come up with more!
   (a) An element of $A_8$ of order 6.
   (b) An isomorphism from $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ to $U(8)$.
   (c) An element of order 6 in $D_7$.
   (d) An element of order 20 in $\mathbb{Z}_{10} \oplus D_4$.
   (e) A proper, non-trivial, normal subgroup of $A_4$.
3. Perform the following calculations or explain why the calculation can't be done. Again, one for each chapter but find more.
   (a) In $S_7$, let $\sigma_1 = (124)(35)$ and $\sigma_2 = (14)(357)$. Find $\sigma_1^2\sigma_2^{-1}$.
   (b) Suppose $\phi \in \mathrm{Aut}(S_5)$ such that $\phi((1234)) = (2453)$. Find $\phi((1432)$.
   (c) Find the left and right cosets of $\langle(1234)\rangle$ of $S_4$ containing $(12)$.
   (d) Find $|(3, (123), V)|$ in $\mathbb{Z}_9 \oplus S_4 \oplus D_4$.
   (e) Make a cayley table for the factor group $\mathbb{Z}_8 \oplus \mathbb{Z}_3/\langle(4, 2)\rangle$.
4. Prove each of the following:
   (a) Suppose $|G| = 27$, show that there exists $a \in G$ such that $|a| = 3$.
   (b) Show that $(G_1 \oplus G_2) \oplus G_3 \cong G_1 \oplus G_2 \oplus G_3$. (Most importantly understand the difference between the left and right side).
   (c) Suppose that $H \trianglelefteq G$ and $a \in G$. Prove by induction that for all $aH \in G/H$ and $n \in \mathbb{N}$, $(aH)^n = a^nH$.