Additional Group Problems Assignment 17

1. Let G be a group. In class we defined $\phi_g : G \to G$ but $\phi_g(x) = gxg^{-1}$.

   (a) In D_4 what is $\phi_D(V)$.

   (b) For each element of $x \in S_3$ find $\phi_{(12)}(x)$.

   (c) Show for $g \in G$, $\phi_g \in \text{Aut}(G)$.

   (d) Show for $g_1, g_2 \in G$, $\phi_{g_1} \circ \phi_{g_2} = \phi_{g_1 g_2}$.

   (e) Show for all $g \in G$ that $(\phi_g)^{-1} = \phi_{g^{-1}}$.

   (f) Show $\text{Inn}(G) \leq \text{Aut}(G)$.

2. Suppose G is a group and $a \in G$ is an element of order 2. Show that $a \in Z(G)$ if and only if $< a > \trianglelefteq G$. (Hint: First figure out what each of these things mean (of course you were going to do this anyway but just in case))