

Additional Problem Assignment 4

Please do each of these problems on a separate sheet of paper, but not necessarily for each part of a problem.

1. Consider a non-square rectangle.
 - (a) Describe all of the symmetries of this shape.
 - (b) Give a Cayley table for these symmetries under composition
 - (c) Does this table form a group? In particular explain how you know it is associative.
 - (d) Is this group Abelian?
2. In class we gave the Cayley table for the Klein 4-group, V (this is the one we called Case I in class where every element was its own inverse). However, we did not complete the proof that it was a group since we did not show it was associative. Show it is associative. (Hint: In mathematics as in life you can often be successful by being persistent or by being clever. I told you could check this by checking 27 equations, but perhaps there is a clever way, perhaps a way using information from this very page. Perhaps...)
3. Using the same approach we did for four-element groups, find all three-element groups. Prove that they are groups (including associativity).