

## Additional Group Problems Assignment 13

1. Here is common logic error that I am seeing. This is an error that I think students are taught is correct in high school so it is very important that we see it is wrong. Suppose you are asked to show that  $m^3 + 1$  and  $m^2$  are relatively prime. Below is an incorrect solution that uses bad logic.

**Solution:** It is enough to show that there exists  $k, l \in \mathbb{Z}$  such that  $k \cdot (m^3 + 1) + l \cdot m^2 = 1$ . We will show that  $k = 1$  and  $l = -m$  works. To see this note that:

$$\begin{aligned} & k \cdot (m^3 + 1) + l \cdot m^2 = 1 \\ \Rightarrow & (1) \cdot (m^3 + 1) + (-m) \cdot m^2 = 1 \\ \Rightarrow & (m^3 + 1) - m^3 = 1 \\ \Rightarrow & 1 = 1 \end{aligned}$$

Thus we have show  $m^3 + 1$  and  $m^2$  are relatively prime, which is what we wanted to show.

- (a) Explain why the above solution is **wrong!**  
(b) Fix the above proof.