Additional Group Problems Assignment 10

1. Consider the group $U(2^m)$ for $m \in \mathbb{N}$ and $m \geq 3$.
   (a) Show that $2^{m-1} + 1$ is in $U(2^m)$.
   (b) Show that $|2^{m-1} + 1| = 2$ in $U(2^m)$.
   (c) Find a different element of $U(2^m)$ that is of order 2. Hint: Look at old homework questions.
   (d) Show that $U(2^m)$ is not cyclic.

2. Show there is no group with exactly two elements of order 2. (Hint: Suppose there are two such elements and show there must be at least one more. Remember though the group may not be abelian. Play with it and don't give up.)

3. (a) Find all subgroups of the cyclic group $\mathbb{Z}_{12}$ (under addition).
   (b) Show that $\mathbb{Z}_{13}^*$ (under multiplication) is cyclic, by showing 2 is a generator.
   (c) Find all subgroups of $\mathbb{Z}_{13}^*$.
   (d) Do you see a similarity between these two groups?