Additional Group Problems Assignment 12

1. Let A, B, C be any sets.
   (a) Show if $f : B \to C$ and $g : A \to B$ are injective then so is $f \circ g : A \to C$.
   (b) Show if $f : B \to C$ and $g : A \to B$ are surjective then so is $f \circ g : A \to C$.
   (c) Show that S_A is closed under composition.

2. Suppose $\alpha, \beta \in S_n$. We say that α and β are disjoint if for all $k, 1 \leq k \leq n$, if $\alpha(k) \neq k$ then $\beta(k) = k$. Intuitively, this means α and β act on disjoint sets.
   (a) Give an example of two elements in S_6 that are disjoint, with neither of them being the identity or cycles.
   (b) Explain why this is a symmetric relationship. I.e. that α and β are disjoint if and only if β and α are disjoint. (Hint: This is as easy as it seems.)
   (c) In class we games a definition of what it means for two cycles to be disjoint. Explain why this definition is consistent. That is show that if two cycles are disjoint under the class definition then they are disjoint under this definition and vice versa.
   (d) Show that for all $\alpha \in S_n$, α and ι (the identity) are disjoint.
   (e) Show that if $\alpha, \beta \in S_n$ are disjoint then so are α and β^{-1}.
   (f) Show that if $\alpha, \beta, \gamma \in S_n$ with α being disjoint from both β and γ, then α is disjoint from $\beta\gamma$.
   (g) Fix $\alpha \in S_n$, show the set of all $\beta \in S_n$ that are disjoint to α form a subgroup of S_n.
   (h) Show that if $\alpha, \beta \in S_n$ are disjoint then $\alpha\beta = \beta\alpha$. (Note: This is the only one that is challenging)