Additional Problem Assignment 8

1. An element a of a group G is said to be an involution if it is its own inverse in G.

   (a) What is the order of a non-identity involution.

   (b) Show that if G is abelian then if a and b are involutions then so is ab. (This was the quiz problem).

   (c) Explain why this proof does not work for a non-abelian group.

   Note: This does not show that that statement is false without the assumption that the group is abelian – it only shows that one particular proof does not work. In order to show that statement is false in general for non-abelian groups you must show a counterexample. You shall do this next.

   (d) Show by giving a particular example of a particular non-abelian group G and two involutions a and b such that ab is not an involution.

   (e) Show for an abelian group G the set of all involutions form a subgroup of G.

2. You have previously shown that if G is a group and H and K are subgroups of G then $H \cap K$ is a subgroup of G. Produce an example of a group G and two subgroups H and K of G such that $H \cup K$ is a group.