Additional Problems Assignment 23

1. Suppose X and Y are topological spaces. Show that $X \times Y$ is Hausdorff if and only if both X and Y are Hausdorff.

2. Suppose X is a set and $\mathcal{T}_1$ and $\mathcal{T}_2$ are topologies on X with $\mathcal{T}_1 \subset \mathcal{T}_2$ (i.e. $\mathcal{T}_1 \subseteq \mathcal{T}_2$ and $\mathcal{T}_1 \neq \mathcal{T}_2$).

   (a) Show if $(X, \mathcal{T}_1)$ is compact and Hausdorff then $(X, \mathcal{T}_2)$ is Hausdorff but not compact.

   (b) Show if $(X, \mathcal{T}_2)$ is compact and Hausdorff then $(X, \mathcal{T}_1)$ is compact but not Hausdorff.

   If you can do this problem in a clever way without a hint then you will be **the** group. You can do it. If you are okay with just being **a** group you can look at this hint.