Exam 1 Review Questions

These questions are designed for you to do to prepare for the exam. Since you are not turning them in the point is not to finish the problems but to really understand them. Of course this is always the point of homework! Don't fool yourself in believing you "kind of" get it.

1. Determine if the following are true or false. Provide either a proof or a counterexample.
   (a) If $f : X \to Y$ is a function and $V \subseteq Y$ is nonempty then $f^{-1}(V)$ is nonempty.
   (b) If $f : X \to Y$ is a function and $A, B \subseteq X$ and $f^{-1}(f(A)) = f^{-1}(f(B))$ then $A = B$.
   (c) If $f : X \to Y$ is a function and $A, B \subseteq X$ then $f(A \cap B) = f(A) \cap f(B)$.
   (d) If $f : X \to Y$ is a function and $A, B \subseteq X$ then $f(A \cup B) = f(A) \cup f(B)$.
   (e) If A is a closed subset of a topological space then $A' \subseteq A$.
   (f) If A is a closed subset of a topological space then $\mathrm{Bd}(A) \subseteq A$.
   (g) If $A = [0, 1)$ then the point 1 is a limit point for A no matter which topology is on $\mathbb{R}$.
   (h) If $A = [0, 1)$ then the point 0 is a limit point for A no matter which topology is on $\mathbb{R}$.
   (i) The set $\mathscr{B} = \{(a, \infty) : a \in \mathbb{Q}\}$ is a base for the topology $(\mathbb{R}, \mathscr{C})$.
   (j) The set $\mathscr{B} = \{[a, b) : a, b \in \mathbb{Q}\}$ is a base for the topology $(\mathbb{R}, \mathscr{H})$.
   (k) In $(\mathbb{R}, \mathscr{U})$ there is a set $A \subseteq \mathbb{R}$ such that $\mathrm{int}(A) = \emptyset$ and $\mathrm{cl}(A) = (0, 1]$.
   (l) In $(\mathbb{R}, \mathscr{U})$ there is a set $A \subseteq \mathbb{R}$ such that $\mathrm{int}(A) = \emptyset$ and $\mathrm{cl}(A) = [0, 1]$.

2. For each of the following $A \subseteq \mathbb{R}$ find $\mathrm{Cl}(A)$, $\mathrm{Int}(A)$, $\mathrm{Bd}(A)$, $\mathrm{Ext}(A)$, and A' for each of the topologies $\mathscr{U}$, $\mathscr{H}$, $\mathscr{C}$, $\mathscr{D}$, and $\mathscr{I}$.
   (a) $A = [0, 1) \cup \{3\}$
   (b) $A = \mathbb{N}$
   (c) $A = ([0, 1] \cup (2, 3]) \cap \mathbb{Q}$
   (d) $A = \{\frac{1}{n} : n \in \mathbb{N}\}$

3. Suppose X is a set and $\mathscr{T}_1$ and $\mathscr{T}_2$ are topologies for X with bases $\mathscr{B}_1$ and $\mathscr{B}_2$ respectively. Show that if $\mathscr{B}_1 \subseteq \mathscr{B}_2$ then $\mathscr{T}_2$ is finer than $\mathscr{T}_1$.

4. For each of the following subspace topologies determine if $[0, 1)$ is open, closed, both or neither.
   (a) $\mathscr{U}_{[-1,1]}$
   (b) $\mathscr{U}_{[0,1]}$
   (c) $\mathscr{H}_{[-1,1]}$
   (d) $\mathscr{H}_{[0,1]}$
   (e) $\mathscr{C}_{[-1,1]}$
   (f) $\mathscr{C}_{[0,1]}$

5. Let $\mathscr{I}$ be the indiscrete topology on $\mathbb{R}$. Let $f : \mathbb{R} \to \mathbb{R}$ be a function.
   (a) Show that f is $\mathscr{I} - \mathscr{U}$ continuous if and only if f is a constant function (a function $f : X \to Y$ is constant if there exists $c \in Y$ such that $f(x) = c$ for all $x \in X$).
   (b) Characterize (with proof) all functions that are $\mathscr{U} - \mathscr{I}$ continuous.