Nick Nobles

Homework

Review Problems

Sec 1.3

Thm 1.3.11 DeMorgan's Law for Indexed Sets
Let $\{A_\alpha \mid \alpha \in \Lambda\}$ be an indexed collection of subsets of a set X. Then
$$X - \bigcup\{A_\alpha \mid \alpha \in \Lambda\} = \bigcap\{X - A_\alpha \mid \alpha \in \Lambda\}.$$

Proof] Let $x \in X$.

$x \in X - \bigcup\{A_\alpha \mid \alpha \in \Lambda\}$

$\Leftrightarrow x \notin \bigcup\{A_\alpha \mid \alpha \in \Lambda\}$

$\Leftrightarrow x \notin A_\alpha \quad \forall \alpha \in \Lambda$

$\Leftrightarrow x \in X - A_\alpha \quad \forall \alpha \in \Lambda$

$\Leftrightarrow x \in \bigcap\{X - A_\alpha \mid \alpha \in \Lambda\}$.

Therefore, $X - \bigcup\{A_\alpha \mid \alpha \in \Lambda\} = \bigcap\{X - A_\alpha \mid \alpha \in \Lambda\}$ ▪

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Homework

Review Problems

Sec 1.4 8. Prove that if $f: X \to Y$ and $g: Y \to Z$ are functions and $g \circ f$ is one-to-one, then f is one-to-one.

Let $x_1, x_2 \in X$ s.t. $f(x_1) = f(x_2)$.

Note: $g \circ f(x_1) = g(f(x_1)) = g(f(x_2)) = g \circ f(x_2)$.

Since $g \circ f$ is one-to-one, $x_1 = x_2$.

Thus f is one-to-one

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Homework

Review Problems

Sec 1.5 Prove $f(\bigcap\{U_\alpha | \alpha \in \Lambda\}) \subseteq \bigcap\{f(U_\alpha) | \alpha \in \Lambda\}$ and give an example where $f(\bigcap\{U_\alpha | \alpha \in \Lambda\}) \neq \bigcap\{f(U_\alpha) | \alpha \in \Lambda\}$.

Proof) Let $y \in f(\bigcap\{U_\alpha | \alpha \in \Lambda\})$.

Then, $\exists x \in \bigcap\{U_\alpha | \alpha \in \Lambda\}$ s.t. $y = f(x)$.

$x \in \bigcap\{U_\alpha | \alpha \in \Lambda\} \Rightarrow x \in U_\alpha \ \forall \alpha \in \Lambda$.

Thus, $f(x) \in f(U_\alpha) \ \forall \alpha \in \Lambda$.

Therefore, $f(x) = y \in \bigcap\{f(U_\alpha) | \alpha \in \Lambda\}$.

Thus, $f(\bigcap\{U_\alpha | \alpha \in \Lambda\}) \subseteq \bigcap\{f(U_\alpha) | \alpha \in \Lambda\}$. ∎

Example)

Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$

Let $U = [-2, -1]$, $V = [1, 2]$.

$f(U \cap V) = f(\emptyset) = \emptyset$.

$f(U) \cap f(V) = [1, 4] \cap [1, 4] = [1, 4]$.

Thus, $f(U \cap V) \neq f(U) \cap f(V)$.

John Landy
5/9/12

Sec. 2.1 Questions

1. True or false? : (a) If a subset V of $\mathbb{R}$ is not equal to a union of open intervals, it can still be open in $\mathcal{U}$.

   (b) Let $\{U_\alpha : \alpha \in \Lambda\}$ be an infinite collection of open subsets of $\mathbb{R}$. Then $\bigcap\{U_\alpha : \alpha \in \Lambda\}$ is an open subset of $\mathbb{R}$.

   (c) Let $\{U_\alpha : \alpha \in \Lambda\}$ be an infinite collection of open subsets of $\mathbb{R}$. Then $\bigcup\{U_\alpha : \alpha \in \Lambda\}$ is an open subset of $\mathbb{R}$.

2. Show $\mathbb{R} - \{a,b,c\}$ is an open set in $\mathcal{U}$ with $a,b,c \in \mathbb{R}$ and $a<b<c$.

3. Show $f(x) = \begin{cases} -1 & \text{if } x<0 \\ 1 & \text{if } x \geq 0 \end{cases}$ is not $\mathcal{U}$-$\mathcal{U}$ continuous.

4. What does it mean for a function to be continuous in terms of open sets?

5. Why is $[a,b)$, with $a,b \in \mathbb{R}$, not open in $(\mathcal{U}, \mathbb{R})$?

Jeremy Leon
Math 385
5/9/2012

Topology Homework: Exam Review Questions

SECTION 2.2 REVIEW QUESTIONS:

1) For a set X, a collection of subsets of X, called $\mathcal{T}$ is a topology if it satisfies the following 3 conditions
   - (a) ____ (a subset of X) $\in \mathcal{T}$ and ____ (a subset of X) $\in \mathcal{T}$.
   - (b) If $V_\alpha \in \mathcal{T}$ for each $\alpha \in \Lambda$, then ________ $\in \mathcal{T}$.
   - (c) If $V_i \in \mathcal{T}$ for $i = 1, 2, \ldots, n$, then ________ $\in \mathcal{T}$.

2) Assume $X = \{1, 2, 3, 4, 5\}$. Explain why the following collections of subsets of X are not topologies on X.
   - (a) $\mathcal{T} = \{X, \emptyset, \{1,2,3\}, \{3,4,5\}\}$
   - (b) $\mathcal{T} = \{X, \{1,2,3\}, \{2,3,4\}, \{2,3\}, \{1,2,3,4\}\}$
   - (c) $\mathcal{T} = \{X, \emptyset, \{1\}, \{2\}, \{1,2\}, \{3\}, \{4\}, \{3,4\}, \{1,2,3,4\}\}$

3.) Prove that the Fort topology is a topology: Let X be infinite with $a \in X$. Then,
$$\mathcal{T} = \{U \subseteq X : a \notin U \text{ or } X - U \text{ is finite}\}$$ is a topology.

SOLUTIONS TO SECTION 2.2 REVIEW QUESTIONS:

1) (a) X, $\emptyset$ or $\emptyset$, X

(b) $\bigcup\{V_\alpha : \alpha \in \Lambda\}$

(c) $\bigcap\{V_i : i = 1, 2, \dots, n\}$

2.) (a) $\{1,2,3\} \in \mathcal{T}$ and $\{3,4,5\} \in \mathcal{T}$ but $\{1,2,3\} \cap \{3,4,5\} = \{3\} \notin \mathcal{T}$.

(b) $\emptyset \notin \mathcal{T}$.

(c) Multiple unions of elements of $\mathcal{T}$ are not open in $\mathcal{T}$:

$\{1\} \cup \{2\} \cup \{3\} = \{1,2,3\} \notin \mathcal{T}$

$\{1\} \cup \{2\} \cup \{4\} = \{1,2,4\} \notin \mathcal{T}$ and others.

The existence of one counterexample is sufficient to prove that $\mathcal{T}$ is not a topology

3.) Let X be infinite with $a \in X$. Then $\mathcal{T} = \{U \subseteq X : a \notin U \text{ or } X - U \text{ is finite}\}$ is a topology.

(a) Let $U = X$. Since $a \in X$, $a \in U$. But $X - U = X - X = \emptyset$, which is finite.

So $X \in \mathcal{T}$

Now, let $U = \emptyset$. Since $a \notin \emptyset$, $\emptyset \in \mathcal{T}$.

(b) Let each U_α for $\alpha \in \Lambda$ be open in $\mathcal{T}$.

If $a \notin U_\alpha$ $\forall \alpha \in \Lambda$, then $a \notin \bigcup\{U_\alpha : \alpha \in \Lambda\}$. So $\bigcup\{U_\alpha : \alpha \in \Lambda\} \in \mathcal{T}$.

If $\exists \alpha_a \in \Lambda$ s.t. $U_{\alpha_a} \in \mathcal{T}$ but $a \in U_{\alpha_a}$, then $X - U_{\alpha_a}$ must be finite.

But $X - \bigcup\{U_\alpha : \alpha \in \Lambda\} \subseteq X - U_{\alpha_a}$ which is finite.

So $X - \bigcup\{U_\alpha : \alpha \in \Lambda\}$ is finite.

Thus, $\bigcup\{U_\alpha : \alpha \in \Lambda\} \in \mathcal{T}$.

(c) Let U_i be open in $\mathcal{T}$ for each $i = 1, 2, \dots, n$.

Thus, if $a \notin U_i$ for some $i = 1, 2, \dots, n$, then $a \notin \bigcap\{U_i : i = 1, 2, \dots, n\}$.

Otherwise if $a \in U_i$ $\forall i = 1, 2, \dots, n$ then $X - U_i$ is finite $\forall i = 1, 2, \dots, n$.

By de Morgan's law, $X - \bigcap\{X - U_i : i = 1, 2, \dots, n\} = \bigcup\{X - U_i : i = 1, 2, \dots, n\}$.

Since $X - U_i$ is finite $\forall i = 1, 2, \dots, n$, then their union, $\bigcup\{X - U_i : i = 1, 2, \dots, n\}$ is finite.

Jonathan Kim
MATH 385
5/9/12

Review problems for Chapter 2.3

2.3.1 Let X = {a, b, c} and let T = {X, ∅, {b}, {a, b}, {b, c}}.

a) List the closed subsets of (X, T)
b) Find Cl({b})
c) Find Cl({a})
d) Find Cl({a, b})
e) Find a proper dense subset of X.

2.3.3 Show that any finite subset of R is U-closed

2.3.8 Let X = R and let T = {U ⊆ X: 1 in U or U = ∅}. Describe the closed subsets of X. Find Cl({1,2}). Is {1, 2} a dense subset of (X, T)? (The collection T is an example of the particular point topology.)

2.3.15 Let A and B be subsets of a topological space (X, T). Show that:

$$X - Cl(A \cup B) = (X - Cl(A)) \cap (X - Cl(B)).$$

Section 2.4: Limit Points, Interior, Boundary, Exterior, More about Closure

- Let (x, T) be a topological space with a subset A of X
- Limit Points: For all elements x of X, x is said to be a [redacted] if [redacted] containing x contain an element of A different from x.
- Interior: The [redacted], noted Int(A), is the set of all points x in X for which there exists an open set U such that x is in U and U is a subset of A.
- Boundary: The [redacted], noted Bd(A), is the set of all points x in X for which all open sets containing x intersect both A and X-A.
- Exterior: The [redacted], noted Ext(A), is the set of all points x in X for which there exists an open set U such that x is in U and U is a subset of X-A.
- Theorem 2.4.4: The set A is closed iff A′ is a subset of A
- Theorem 2.4.6: The set AUA′ is closed
- Theorem 2.4.7: Cl(A) = AUA′
- Theorem 2.4.9: x is in Cl(A) iff for any open set U containing x, $U \cap A \neq \emptyset$
- Theorem 2.4.14: Int(A) = U{U subset of A: U is an open set}
- Theorem 2.4.15: Int(A) is an open set
- Theorem 2.4.16: A is open iff A = Int(A)
- Theorem 2.4.18: Ext(A) = Int(X-A)
- Theorem 2.4.21: The sets Int(A), Bd(A), and Ext(A) are pairwise disjoint and X = Int(A)UBd(A)UExt(A)
- Fact about Closure: Cl(A) = Int(A)UBd(A)

✓ What is Ext(Ext(A))?

✓ How does Cl(A) relate to Ext(A)?

✓ Let A = [2,5] U (-1,1) U {9,10} . What is Cl(A)? Bd(A)? Ext(A)? A′? Int(A)?

✓ Let X = {a,b,c,d,e,f,g,h}, T = {X, Ø, {a,b,c},{c,d}, {c}, {e}, {a,b,c,h}, {a,f,g}, {f}}. Let A = {a,d,g,f,h}. What is Int(A)? Bd(A)? Ext(A)? A′? Cl(A)?

2.5 Basic Open Sets

① Explain why each of the following is not a base for a topology on $\mathbb{R}$.

a) $\{(n, n+1) : n \in \mathbb{Z}\}$

b) $\{(x-1, x+1) : x \in \mathbb{R}\}$

c) $\{(x, x+1) : x \in \mathbb{R}\}$

② Give basic open sets for $\mathcal{H}$, $\mathcal{C}$, + $\mathcal{U}$.

③ If X + Y are homeomorphic and $f : X \to Y$ is a homeomorphism and $\mathcal{B}$ is a base for X. Is $f(\mathcal{B})$ a base for Y?

Katherine Malewski
Math 385, Dr. Parker

Section 3.1 Review Problems

① Let $A = (2,7]$ be a subspace of $(\mathbb{R}, \mathcal{H})$. Which of the following sets are $\mathcal{H}_A$-open?

(a) $(2,5)$
(b) $[4,7)$
(c) $(2,5]$
(d) $[3,4]$
(e) $\emptyset$
(f) $[3,7)$
(g) $(2,3)$

② Prove that if $(X, \mathcal{T})$ is a topological space with $A \subseteq X$ and $U \subseteq A$, then $A \cap \mathrm{Int}_X(U) \subseteq \mathrm{Int}_A(U)$

③ Prove that the collection $\mathcal{T}_A = \{U \cap A : U \in \mathcal{T}\}$ is a topology for the set A.

Ashley Klahr

3.2 Continuity

1. Let $\mathcal{I}$ be the indescrete topology on $\mathbb{R}$. Let $f:\mathbb{R}\to\mathbb{R}$ be a function.
   a) Show that f is $\mathcal{I}$-$\mathcal{U}$ cts iff f is a constant function
   b) Characterize (with proof) all functions that are $\mathcal{U}$-$\mathcal{I}$ cts.

2. Which of the following are (i) $\mathcal{U}$-neighborhoods of 1 (ii) neighborhoods of 1
   a) $[0,2]$ b) $[0,1]$ c) $[0,1)$
   d) $[1,2)$ e) $\{1,2\}$ f) $(0,1]$

3. Determine ~~which of the following~~ if the function $f:\mathbb{R}\to\mathbb{R}$ given by $f(x)=\begin{cases}-1 & \text{if } x>0\\ 1-x & \text{if } x\le 0\end{cases}$
   a) $\mathcal{U}$-$\mathcal{U}$ cts b) $\mathcal{U}$-$\mathcal{H}$ cts c) $\mathcal{U}$-$\mathcal{C}$ cts
   d) $\mathcal{H}$-$\mathcal{U}$ cts e) $\mathcal{C}$-$\mathcal{C}$ cts f) $\mathcal{C}$-$\mathcal{U}$ cts
   g) $\mathcal{C}$-$\mathcal{H}$ cts h) $\mathcal{H}$-$\mathcal{H}$ cts

4. Prove: let $(X,\mathcal{J}_1)$ + $(Y,\mathcal{J}_2)$ be a topological spaces + $\mathcal{B}$ be a base for $(Y,\mathcal{J}_2)$. Then the following are equivalent ~~base~~
   a) f is $(\mathcal{J}_1$-$\mathcal{J}_2)$ cts
   b) For all closed $F\subseteq Y$, $f^{-1}(F)$ is closed in X.
   c) $\forall B\in\mathcal{B}$, $f^{-1}(B)$ is open in X
   d) $\forall x\in X$ and nbh of $f(x)$, $f^{-1}(N)$ is a nbh of X
   e) $\forall x\in X$ and N nbh of $f(x)$ $\exists$ nbh M of x st $f(M)\subseteq N$
   f) If $A\subseteq X$ then $f(Cl(A))\subseteq Cl(f(A))$

Chris Davidson

Problems for section 3.3

1) Which of the following are homeomophic?

If it is, define a function.

a) $X = \{a,b,c\}$ $\mathcal{T} = \{X, \phi, \{c\}, \{a,b\}\}$

$Y = \{x,y,z\}$ $\mathcal{S} = \{Y, \phi, \{x\}, \{x,y\}\}$.

Is a homeomorphism, let $f:(X,\mathcal{T}) \to (Y,\mathcal{S})$ be defined by $f(a)=x$, $f(b)=y$, $f(c)=z$.

b) $X = \{a,b,c\}$ $\mathcal{T} = \{X, \phi, \{a\}, \{a,b\}\}$

$Y = \{x,y,z\}$ $\mathcal{S} = \{Y, \phi, \{x\}, \{y,z\}\}$

Not homeomorphic.

2) Let $(X,\mathcal{T})$, $(Y,\mathcal{S})$, $(Z,\mathcal{V})$ be top spaces

If $g: X \to Y$ is a homeomorphism and $f: X - Z$ is a homeomorphism, then $h: Y \to Z$ is a homeomorphism.

Since $g: X \to Y$ is a homeomorphism $g = g^{-1}: Y \to X$ is a homeomorphism

Since $g: Y - X$ is a homeomorphism and $f: Y - X$ is a homeomorphism, $h = g \circ f: Y - Z$ is a homeomorphism, as desired.

3) Prove that the spaces $((1,\frac{3}{2}), \mathcal{U}_{(0,1)})$ and $((2,4), \mathcal{U}_{(2,4)})$ are homeomorphic.

Let $f:(1,\frac{3}{2}) \to (2,4)$ be given by $f(x) = 4x - 2$

4.1 : Product Spaces

Kyle Warren

Important theorem's

1) Let $(X, \mathcal{T})$ & $(Y, \mathcal{S})$ be top spaces. The collection $\mathcal{B} = \{ U \times V : U \in \mathcal{T}, V \in \mathcal{S} \}$ forms a base for a topology on $X \times Y$

2) //same goes for basic open sets //
   If $\mathcal{B}_1$ & $\mathcal{B}_2$ are bases for $\mathcal{T}_1$ and $\mathcal{T}_2$, then the collection $\mathcal{B} = \{ B_1 \times B_2 : B_1 \in \mathcal{B}_1, B_2 \in \mathcal{B}_2 \}$ is a base for product topology on $X \times Y$.

3) Let $(X, \mathcal{T})$ and $(Y, \mathcal{S})$ be top spaces with $A \subseteq X$, $B \subseteq Y$. Then $Cl(A \times B) = Cl(A) \times Cl(B)$.

4) Let $(X, \mathcal{T})$ and $(Y, \mathcal{S})$ be top spaces with $A \subseteq X$, $B \subseteq Y$. Then $Int(A \times B) = Int(A) \times Int(B)$

Practice Problems

1. Let $X = \mathbb{R}$ have the $\mathcal{U}$-topology and $Y = \mathbb{R}$ have the $\mathcal{H}$-topology. Describe a typical open subset of $X \times Y$

2. Let $X = \{a, b, c\}$, $\mathcal{T} = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$, $Y = \{p, q\}$, $\mathcal{S} = \{Y, \emptyset, \{q\}\}$
   List the sets in the base for the product topology on $X \times Y$ given in theorem 1 above.

3. Let $A = (4, 5] \times (4, 5)$. Find $Cl(A)$ on the product topology $\mathcal{H} \times \mathcal{U}$.

1

Carefully define a *product space.* What is the base for a product space?

2

Let $p_i : X_1 \times X_2 \times \cdots \times X_n \to X_i$ be the *ith* projection function. When is p_1 a homeomorphism? p_2? When is p_j a homeomorphism for all $1 \leq j \leq n$? Prove it. Give a simple reason why the projection function is not normally a homeomorphism.

3

Suppose $\mathcal{B}$ is a base for $X_1 \times X_2 \times \cdots \times X_n$. Construct a base for X_i.

4

Let $X = Y = \mathbb{R}$, all with the usual topology, $p_y : X \times Y \to Y$ be the projection function onto Y and $f : X \to X \times Y$ such that $f(x) = (x, \sin(x))$. Let

$$A = (p_y \circ f)(0, 2\pi).$$

Is A with the usual relative topology normal?

Alex Hill

5/9/12

5.1 Review Questions

1. State whether or not the following is true or false. If true prove it if false provide a counterexample:

a) Any discrete topological space is disconnected.

b) The least upper bound of a set that is bounded above is unique.

c) Any bounded set is connected in $\mathcal{U}$.

d) If X can have topologies $\mathcal{T}$ and $\mathcal{J}$ such that $\mathcal{J}$ is finer than $\mathcal{T}$, then it is possible for $(X,\mathcal{T})$ to be connected and $(X,\mathcal{J})$ disconnected.

2. Let $X=\mathbb{R}$ and $\mathcal{T}=\{U\subseteq X : U=\emptyset \text{ or } X\setminus U \text{ is finite}\}$. Prove or disprove $(X,\mathcal{T})$ is connected.

3. Let $X=[0,1]\cup\{4\}$. Show that $(X,\mathcal{U}_X)$ is disconnected.

4. Show that a topological space X is disconnected iff $\exists U$ that is nonempty, open, and closed in X.

5.2

Ryan O'Halloran

1. let A be a proper subset of X and let B be a proper subset of Y. If X and Y are connected, show that $(X \times Y) - (A \times B)$ is connected.
Hint: look at proof of Thm. 3.3.3
Dr. Parker has authorized bonus points for a complete solution

2. let $\mathcal{T}_1$ and $\mathcal{T}_2$ be two topologies on X. If $\mathcal{T}_2$ is finer than $\mathcal{T}_1$, what does connectedness on X in one topology imply about connectedness in the other.

3. Give an example of a family of connected sets whose union is not connected.

4. Give an example to show that the intersection of two connected sets is not necessarily connected.

Griselda Resendiz
Topology
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Review Problems 5.3

1. Determine if the following subsets of $(\mathbb{R}^2, \mathcal{U})$ are connected. If so, prove it. If not draw two open subsets that disconnect them

a. $((2,5) \times (4,5)) \cup ((3,7) \times (3,6))$

b. $((3,0) \times (2,3)) \cup ((1,2) \times (0,1))$

2. Are the following subspaces of $\mathbb{R}$ homeomorphic? Explain

$X = (0,1) \cup (5,8) \cup \{9\}$
$Y = (-1,0) \cup (3,4) \cup (7,9)$
$Z = (2,3) \cup (4,7) \cup \{0\}$
$W = (-1,0) \cup (3,6)$

a. $X \approx Y$

c. $X \approx Z$

b. $Y \approx W$

d. $Y \approx Z$

3. let X be a topological space and let $Y = \{0,1,2\}$ have the D topology. Assume $f: X \to Y$ is continuous function. If A is a connected subset of X, What are the possible values of the image $f(A)$? Explain.

6.1 Compactness Problems

1. Let X be a set under the finite complement topology. Find all compact subsets of X.

2. Prove that $A \subseteq \mathbb{R}$ is compact iff every subset of A has an infimum & supremum in A.

3. Let $X = [0,1]$ and $\tau = \{ \mathcal{U} \in \tau : \mathcal{U} = \emptyset, \mathcal{U} = X, \text{ or } \mathcal{U} \subseteq (0,1) \}$ where $\mathcal{U}$ is open in the usual topology on $\mathbb{R}$. Show (X, τ) is compact but not Hausdorff.

4. Let $X = \mathbb{R}$ have the usual topology and define $\tau \subseteq \mathcal{U}$ where $\tau = \{ \mathcal{U} \in \mathcal{U} : \text{if } 0 \in U, \exists\, \varepsilon > 0 \text{ s.t. } (-\infty, -\delta) \cup (\delta, \infty) \subseteq \mathcal{U} \}$. Prove $(\mathbb{R}, \tau)$ is compact & Hausdorff.

Marlowe Quart Section 6.2

True or False: if false, give a counter example.

① $(\mathbb{R}, \mathcal{U}) \approx ([0,1], \mathcal{U}_{[0,1]})$ is a homeomorphism

② $(\mathbb{R}, \mathcal{U}) \times ([0,1], \mathcal{U}_{[0,1]})$ is compact

③ for $X = \mathbb{R}$, $\tau = \{U \subseteq X : U = X \text{ or } 1 \notin U\}$ is compact $\Rightarrow$ $Y = \mathbb{R}$, $\tau = \{U \subseteq Y : U = \emptyset \text{ or } 1 \in U\}$ is compact.

for H & C topologies, determine if the following are compact:

a) $[0,1] \times [0,100]$ b) $(0,1) \times [1,2]$

c) $\{0\} \times ([1,2] \cup [5,8])$ d) $[0,\infty) \times [0,\infty)$

e) $[0,\infty) \times [1,2]$ f) $[1,2) \times [1,2)$

7.1 review problems. Marc Slaughter

1) Recall the finite complement topology:

$\tau = \{A \subseteq X \mid A = \emptyset \text{ or } X/A \text{ is finite}\}$

Determine with proof whether $(\mathbb{R}, \tau)$ is T_i for $i = \{0, 1, 2\}$

Hint: Start with T_2 since $T_2 \Rightarrow T_1 \Rightarrow T_0$

2) True or false: (with proof or counterexample)

Let $(\mathbb{R}, \tau)$ be a topological space with $\tau = (U \subseteq \mathbb{R} : 1 \in U \text{ or } U = \emptyset\}$

a) $(0,2) \times (0,1)$ is T_2 with the τ relative topology

b) $(0,2) \times (0,1)$ is T_1 " " " "

c) $(0,2) \times (0,1)$ is T_0 " " " "

3) Let X be a Hausdorff Topological space.

We know that $X \times X$ is Hausdorff by Thm 7.1.0

Determine whether the set $A = \{(x,x), x \in X\} \subseteq X \times X$

is closed.

Section 7.2-7.3

1. Let $\mathcal{T}$ be a topology on $\mathbb{R}$ such that $\forall U \in \mathcal{T}$, and for any $x \in U$ $\exists \epsilon > 0$ where $[x, x+\epsilon) \subset U$, let us denote this as $\mathbb{R}^s$. When you take $\mathbb{R}^s x \mathbb{R}^s$ it is called the Sorgenfrey Half–open square topology. Is $\mathbb{R}^s$ normal? Is the Sorgenfrey Half–open square topology normal? If either or both is prove. If they are not give a counterexample.

2. Prove: A space X is regular iff for each $x \in X$, the closed neighborhoods of x form a basis of neighborhoods of x.

3. Give examples of the following topologies on the set $X = \{a, b, c, d\}$ (not the discrete or indiscrete topologies).

   (a) a topology $\mathcal{T}$ such that $(X, \mathcal{T})$ is T_0 space but not T_1 space.

   (b) a topology $\mathcal{S}$ such that $(X, \mathcal{S})$ is T_1 space but not T_2 space.

   (c) a topology $\mathcal{R}$ such that $(X, \mathcal{R})$ is T_2 space but not T_3 space.

   (d) a topology $\mathcal{Q}$ such that $(X, \mathcal{Q})$ is regular but not normal

   (e) a topology $\mathcal{P}$ such that $(X, \mathcal{P})$ is normal

4. Prove that T_4 property is a topological property.

5. True or False. Prove the true problems and find a counterexample for false.

   (a) Any indiscrete topological space is not Hausdorff

   (b) Closed subsets of normal spaces are normal

   (c) Every T_4 space is regular

   (d) Every T_i space is a T_{i-1} space for each $i \in \{1, 2, 3, 4, 5\}$

   (e) If a topological space does not have any nonempty disjoint open sets, then the space is not normal.