**Hittin’ The Spot**

**Abstract**

We consider the “sweet spot” on a baseball bat, focusing on its location as well as the varying effects it provides on bats of different materials. Contrary to the idea that the “sweet spot” is located at the point which provides maximum torque with respect to a hitters hands—the end of the bat—the sweet spot is found with respect to the bat’s center of mass. By allowing the ball to contact the bat at a point which provides a torque of equal magnitude but opposite direction as the torque produced by a hitter’s hands, the greatest amount of momentum is carried into the ball.

By corking a baseball bat, the center of mass is altered. In accordance with the torque relationship we found to represent the ball-to-bat collision, this gives a hitter an advantage in terms of bat speed and “sweet spot” location. However, such a procedure does not enhance the effect of the “sweet spot.” In contrast, the material out of which a bat is constructed may substantially alter the effect of the “sweet spot.” Since the ball-to-bat collision may be modeled as a relationship of energy conversions, how a material affects these conversions is quite pertinent. Therefore, one material, such as aluminum, may help accelerate the ball off the bat while another, such as wood, may slow the ball upon leaving the bat.

**Initial Assumptions**

* The bat’s “sweet spot” can be modeled in 1-D.
* We will be attempting to maximize velocity of the ball off the bat.
* The weight of an MLB baseball is 0.145 kg.
* During collision, a baseball stays on the bat for 0.1 ms.
* The force applied by a batter’s hands is perpendicular to the bat.
* Angular momentum of the ball is negligible.

**Location of the Sweet Spot**

At first mathematical inquiry the “sweet spot” of a baseball bat should be at the end of the baseball bat, due to the fact that this point of the bat has the most momentum. However, we know from baseball history that this is not the case. This problem has been approached in many ways, and exhaustive research has been done to explain this phenomenon. We will approach the problem of the baseball bat “sweet spot” from a conservation of momentum perspective. In a baseball-to-bat collision, the goal is to transfer as much momentum as possible from the original collision into the ball. The ball is on the bat for 0.1ms during the collision. Due to this short time period that the ball is in contact with the bat we can consider the initial collision to be based on linear momentum. Therefore we arrive with Equation 1.

$$\left(m\*v\_{i}\right)+\left(M\*V\_{i}\right)=\left(m\*v\_{f}\right)+\left(M\*V\_{f}\right)+\left(I\_{bat}\*w\_{bat}\right)+\left(I\_{ball}\*w\_{ball}\right)$$

**Equation 1: Conservation of Momentum from Collision**

*m* = Mass of Baseball

*M* = Mass of Baseball Bat

*Vi* = Initial Velocity of Bat

*vi* = Initial Velocity of Ball

*Vf* = Final Velocity of Bat

*vf* = Final Velocity of Baseball

*Ibat*= Moment of Inertia of Bat

*wbat*= Angular Frequency of Bat after Collision

*Iball*= Moment of Inertia of Ball

*wbat* = Angular Frequency of Ball after Collision

 The angular momentum of the ball ($I\_{ball}\*w\_{ball}$) relative to other terms will be minimal; therefore, we will ignore it. To transfer the most momentum to the ball we must eliminate the angular momentum of the bat after collision. Therefore when this term $\left(I\_{bat}\*w\_{bat}\right)$ equals 0, the most momentum is carried into the ball. This situation results in the highest possible ball velocity.

 We must find a way to eliminate the bat angular momentum term of Equation 1. We must balance the torques placed on the bat during the collision to nullify the angular momentum. We are only interested in the dimension of the plane of the swing. Therefore, we can assume 1 dimension for our calculations. When we equate the torques created by the equation we get Equation 2.

$$\sin(90°\*a\*F\_{hands on bat}=F\_{ball}\*b\*)\sin(θ\_{collsion})$$

**Equation 2: Torques on bat in Collision**

*a* = distance from the point a player exerts force on the bat and the Center of Mass

*b* = distance from the point that the ball exerts its force on the bat and the Center of Mass.

*θcollision* = Angle at which the ball strikes the bat.

*Fhands on bat*= Force exerted by the player on the bat.

*Fbal*l = $(\frac{d}{dt}\left(m \_{ball}\*v\_{ball}\right))=$ Force exerted by the ball on the bat.

 We calculate *a* and *b* based on the Center of Mass, because the bat will rotate around its Center of Mass due to a force. Therefore, *a* and *b* can be substituted for r when solving for the Torque on the bat (*Torque = r \*sin(θforce)\*F*). This gives a reference point for where the sweet spot is located on the bat, and allows us to solve for *b*. Solving for *b* lends the location of the “sweet spot”, and Equation 3.

$$b=\frac{\sin(90°\*a\*F\_{hands on bat})}{(\frac{d}{dt}\left(m \_{ball}\*v\_{ball}\right))\*\sin(θ))}$$

**Equation 3: Sweet Spot Calculation**

 This equation relates the forces that the batter and the ball apply to the bat and where the “sweet spot” will be located based on those forces. When logically thinking about this equation we come to the conclusion that it also satisfies our empirical knowledge. If the ball is pitched at a fast pace, the denominator will be larger bringing b (the sweet spot) closer to the batter. Where, if the ball is lobbed in very slowly b is pushed farther away from the batter. Many baseball players claim that when the “sweet spot” is hit they can barely feel the strike of the ball. This is shown in our example because the bat flows smoothly through the ball with no added angular momentum on the bat. Figure 1 showcases the dimensions of the bat, and how they relate to Equation 3.



Figure 1: Sweet Spot of the Bat

**Corking of a Bat**

Many players in the MLB have been known to cork their bats to create a perceived advantage for themselves while batting. This begs the question what type of advantage is created by the corking of the bat. First, the effect that corking a bat has on the “sweet spot” of the bat. A bat is usually corked by hollowing out 1” of material out of the middle of the bat starting at the end and going 12” into the bat. This has an effect of bringing the Center of Mass closer to the handle of the bat and therefore changing the position of *b* in Equation 3. Therefore, corking of the bat effectively moves the “sweet spot” in closer to the batter.

The movement of the “sweet spot” would not be a sufficient enough reason for a player to cork their bat. The major advantage of corking a bat would be to generate bat speed. Force applied to swing a bat is equal to the moment of inertia multiplied by the angular acceleration of the bat, and is written as Equation 4.

$$F=I\*(\frac{dw}{dt})$$

**Equation 4: Force applied to Bat**

*F* = Force

*I* = Moment of Inertia

*w* = Angular Velocity

$\frac{dw}{dt}$ = Angular Acceleration

 Corking of the bat lowers *I,* allowing for a greater value of $\frac{dw}{dt}$, resulting in greater bat speed. Therefore, corking a bat allows the player to more effectively control his bat, and generate more bat speed for hitting the ball. However, going back to Equation 1, we can see that even though we raise the velocity of the bat on impact, we also proportionally lower the mass of the bat. A corked bat does not enhance the “sweet spot” effect of the bat because of this loss in bat mass. The reason that corking is outlawed in the MLB is surely the increased bat speed and control one can gain by corking of the bat. If a player can wait a split-second more before he decides to swing at a pitch that is a decided advantage for that player. This is the reason that corked bats are not allowed in baseball.

**Wood Bat vs. Aluminum Bat**

Assume that all contact between the ball and bat is done on the sweet spot. Therefore, based on Equation 1 we can negate the angular momentum of the bat after the collision. This assumption leaves us with a linear collision of two directly opposing forces. We also assume that all aspects of the collision such as bat speed and ball speed remain constant and the only thing that is changing is the material of the bat. The speed of the bat through the ball will remain constant for both the wood and aluminum bat. This will transfer kinetic energy to the ball that will be constant for the wood and aluminum bats, and can be ignored in analysis. Therefore, we will analyze the difference in the aluminum and wood bat collisions based on the energy lost in ball compression and bat compression. The most effective way to analyze this scenario is using the model of a spring for the bat and the ball. This leads us to Equation 5.

$$U\_{ball compression}+U\_{bat compression}=KE\_{bat recoil}+KE\_{ball}+U\_{ball compression}$$

**Equation 5: Conservation of Energy into Ball**

$U\_{ball compression}$ **=** $\frac{1}{2}k\_{ball}\*x^{2}$ = Potential Energy of the Ball Compression

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$KE\_{bat recoil}$ **=** $\frac{1}{2}m\_{bat}\*v\_{bat}^{2}$ **=** Kinetic Energy of the Bat

$KE\_{ball recoil}$ **=** $\frac{1}{2}m\_{ball}\*v\_{ball}^{2}$ **=** Kinetic Energy of the Ball

 The left portion of this equation represents when all of the original kinetic energy of the ball and bat have been transferred into potential energy using Hooke’s Law. The right side of the equation represents the instant that the ball is no longer in contact with the bat. We have shown that $U\_{ball compression}$is present on both sides of this equation. This is because the ball has a much lower dynamic transverse elasticity than the bat of either material. Therefore, because of this quality we can assume the ball does not decompress while still on the bat. This means that the energy will be lost in the $U\_{ball compression}$concerning the final kinetic energy of the ball. The aluminum bat is hollow and therefore has a higher amount of energy stored in the bat compression. The thin aluminum barrel also allows the bat to compress a significant amount. This is contrasted by the thickness of a solid wooden bat, which provides very little bat compression. Furthermore, aluminum has a modulus of elasticity of 6.9e10 N/m2 which is nearly seven times greater than that of white ash (1.2e10 N/m2). These factors combine to allow an aluminum bat to recoil at a much faster and powerful rate, resulting in an increased kinetic energy for the ball. Conversely, the wood bat has much greater potential energy stored in the ball that is lost to the kinetic energy of the ball.

**Conclusions**

 Upon analyzing the “sweet spot” on a baseball bat, we developed several conclusions based on the models we created for the ball-to-bat collision. First, the location of the “sweet spot” is based on the relationship of the torques produced on opposing sides of the bat’s center of mass. By equalizing the magnitudes of these torques, the most solid contact is produced and yet a batter may feel as if no contact occurred as the torques negate each other’s influence. Next, the affect of corking a bat is advantageous since it moves the “sweet spot” closer to a hitter’s hands. As the “sweet spot” slides in with the bat’s center of mass, a batter benefits from a lighter bat and a more accurate swing. However, this does not enhance the ball-to-bat collision or the effect of the “sweet spot.” Lastly, the material of the bat plays a large role on the actual effect of the “sweet spot.” Aluminum, which allows for greater compression and faster recoil, helps increase the kinetic energy of the ball as it leaves the bat. In contrast, a solid wood bat forces the ball to compress, providing a lower kinetic energy as the ball-to-bat collision ends.

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