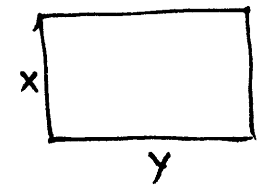
Sample Homework Write-Up – Excellent

Problem: A farmer has 200 feet of fencing to make a rectangular pen. What are the length and width of the largest area pen that she can make with her fencing?



Let x = width of pen, in ft
y = length of pen, in ft

Constraint : $2x + 2y = 200$
$\Rightarrow y = 100 - x$

<u>Want to Maximize Area</u>; Let A = area of pen, in ft^2.

$$A = x \cdot y$$
$$= x(100 - x)$$
$$= 100x - x^2$$

<u>Find critical points</u>: x-values where $A' = 0$ or A' DNE

$A' = 100 - 2x \Rightarrow A'$ exists everywhere,

$A' = 0 \Rightarrow 0 = 100 - 2x$
$\Rightarrow x = 50$

<u>Check if $x = 50$ is max</u> using second derivative test

$A'' = -2 < 0 \Rightarrow A$ is always concave down
$\Rightarrow x = 50$ is a max

<u>Find y</u>

$y = 100 - x$
$= 50.$

The farmer should build her pen with length 50 ft and width 50 ft in order to maximize area.

Sample Homework Write-Up – Good

Problem: A farmer has 200 feet of fencing to make a rectangular pen. What are the length and width of the largest area pen that she can make with her fencing?

$x =$ width of pen
$y =$ length of pen

$2x + 2y = 200 \Rightarrow y = 100 - x$

<u>Maximize area</u>

$$\begin{aligned} A &= x \cdot y \\ &= x(100 - x) \\ &= 100x - x^2 \end{aligned}$$

$A' = 100 - 2x$

$\underline{A' = 0}$ $\quad$ A' exists everywhere

$100 - 2x = 0$

$x = 50$

$A'' = -2 < 0$ $\quad$ concave down $\Rightarrow x = 50$ is max

$$\begin{aligned} y &= 100 - x \\ &= 50 \end{aligned}$$

Pen should be $50\text{ft} \times 50\text{ft}$.

Sample Homework Write-Up – Barely Acceptable

Problem: A farmer has 200 feet of fencing to make a rectangular pen. What are the length and width of the largest area pen that she can make with her fencing?

$$2x+2y=200$$

$$x+y=100 \qquad y=100-x$$

$$A=x\cdot y$$

$$A=x(100-x)$$

$$A=100x-x^2$$

$$A'=100-2x=0$$

$$x=50$$

$$A''=-2<0 \qquad \text{cu}$$

$$y=50$$

$$\boxed{50\text{ft}\times50\text{ft}}$$

Sample Homework Write-Up – Ugly / Unacceptable

Problem: A farmer has 200 feet of fencing to make a rectangular pen. What are the length and width of the largest area pen that she can make with her fencing?

$2x+2y=200$

$2y = 200-2x$
$y = 100-x$

$(100-x)\cdot x$

$100x-x^2$

$100-2x \qquad x=50$

$y=50$

$$\begin{array}{r} 100 \\ -\ 50 \\ \hline 50 \end{array}$$

-2 c.u.

50 ft × 50 ft