Trigonometric Integrals Review (Section 6.2)

Motivating example

- $\int \sin^2 x \cos x \, dx$ – by substitution
- $\int \cos^3 x \, dx$

Trig Identities which may be useful

- $\cos^2 \theta + \sin^2 \theta = 1$
- $1 + \tan^2 \theta = \sec^2 \theta$
- $\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$
- $\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$
- $\sin 2\theta = 2 \sin \theta \cos \theta$

Form $\int \sin^m x \cos^n x \, dx$

- If any power is odd, use trig identity (a): $\int \cos^3 x \sin^4 x \, dx$
- If both powers are even, use identities (c) and (d): $\int \sin^4 x \, dx$
- $\int \sin^2 x \cos^2 x \, dx$

Form $\int \tan^m x \sec^n x \, dx$

- If sec has even exponent, save factor of $\sec^2 x$: $\int \tan^4 x \sec^4 x \, dx$
- If tan has odd exponent, save factor of $\sec x \tan x$: $\int \tan^5 x \sec^9 x \, dx$
- $\int \tan^5 x \, dx$
- $\int (\tan x + \cos x)^2 \, dx$

Basic antiderivatives to know:

1. $\int \tan x \, dx = \ln |\sec x| + C$
2. $\int \sec x \, dx = \ln |\sec x + \tan x| + C$ (multiply top and bottom by $\sec x + \tan x$)

Trig Substitution

- Motivating example: $\displaystyle\int_0^3 \frac{dx}{\sqrt{4 + x^2}}$
- Getting rid of square root of sum or difference of squares: use one of
  - $\cos^2 \theta + \sin^2 \theta = 1$
  - $1 + \tan^2 \theta = \sec^2 \theta$
- Note: Don't forget to convert back to original variable!
- Use trig substitution for integrals containing $\sqrt{a^2 - x^2}$,$\sqrt{a^2 + x^2}$, $\sqrt{x^2 - a^2}$

Examples

1. $\int \sqrt{9 - x^2} \, dx$
2. $\displaystyle\int \frac{\sqrt{25x^2 - 4}}{x} \, dx$
3. $\displaystyle\int_0^{\frac{3\sqrt{3}}{2}} \frac{x^3}{(4x^2 + 9)^{3/2}} \, dx$
4. $\displaystyle\int \frac{dx}{\sqrt{x^2 + 4x + 8}}$
5. $\displaystyle\int \frac{\sqrt{x^2 - 9}}{x^3} \, dx$