Math 250 Polar Integral Practice

Polar Integrals

1. Evaluate $\int_R \sin(x^2+y^2) \;\; dA$ where R is the disk of radius 2 centered at the origin. $(\pi(1-\cos 4))$

2. Evaluate $\int_R x^2 - y^2 \;\; dA$, where R is the first quadrant region between the circles of radius 1 and radius 2. (0)

3. Consider the integral $\int_0^3 \int_{x/3}^1 f(x,y) \;\; dy \;\; dx$
   (a) Sketch the region R over which the integration is being performed
   (b) Rewrite the integral with the order of integration reversed.
   (c) Rewrite the integral in polar coordinates.

4. Convert the following integrals to polar coordinates and evaluate:
   (a) $\int_{-1}^0 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x \;\; dy \;\; dx$ (-2/3)
   (b) $\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} xy \;\; dx \;\; dy$
   (c) $\int_0^{\sqrt{6}} \int_{-x}^{x} \;\; dy \;\; dx$ (6)

5. * An ice cream cone can be modeled by the region bounded by the hemisphere $z = \sqrt{8-x^2-y^2}$ and the cone $z = \sqrt{x^2+y^2}$. Find its volume. $(32\pi(\sqrt{2}-1)/3)$