Calc 3 Cylindrical and Spherical Integral Practice

For all these problems, you must use either spherical or cylindrical coordinates.

1. Sketch the region over which the integration is being performed: $\int_0^{\pi/2}\int_{\pi/2}^{\pi}\int_0^1 f(\rho,\theta,\phi)\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

2. * Evaluate $\iiint_R y\,dV$, where R is the solid that lies between the cylinders $x^2+y^2=1$ and $x^2+y^2=4$, above the x,y plane, and below the plane $z=x+2$. (0)

3. Evaluate $\iiint_R xe^{(x^2+y^2+z^2)^2}\,dV$, where R is the solid that lies between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$ in the first octant. $(\frac{\pi}{16}(e^{16}-e))$

4. Evaluate $\iiint_R x^2\,dV$, where R is the solid that lies within the cylinder $x^2+y^2=1$, above the plane $z=0$, and below the cone $z^2=4x^2+4y^2$. $(2\pi/5)$

5. Find the volume of the solid that lies above the cone $\phi=\pi/3$ and below the sphere $\rho=4\cos\phi$. $(10\ \pi)$

6. Find the mass of the solid bounded above by the hemisphere $z=\sqrt{25-x^2-y^2}$ and below by the plane $z=4$ where the density at a point P is inversely proportional to the distance from the origin. [Hint: Express the upper ϕ limit of integration as an inverse cosine.] (πk)

7. Challenge: Find the mass of the solid bounded below by the $x-y$ plane, on the sides by the hemisphere $z=\sqrt{25-x^2-y^2}$, and above by the plane $z=4$, where the density at a point P is inversely proportional to the distance from the origin. [Hint: Express the upper ϕ limit of integration as an inverse cosine.] ()

8. Evaluate the integrals below by changing to either spherical or cylindrical coordinates, whichever is more appropriate.

   (a) * $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{2-x^2-y^2}(x^2+y^2)^{3/2}\,dz\,dy\,dx$ $(8\pi/35)$

   (b) $\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{0}^{\sqrt{9-x^2-y^2}} z\sqrt{x^2+y^2+z^2}\,dz\,dy\,dx$ $(243\pi/5)$

   (c) $\int_0^1\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\frac{1}{\sqrt{x^2+y^2}}\,dy\,dx\,dz$ (2π)

   (d) $\int_0^1\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-\sqrt{1-x^2-z^2}}^{\sqrt{1-x^2-z^2}}\frac{1}{\sqrt{x^2+y^2+z^2}}\,dy\,dz\,dx$ (π)