Math 250 Line Integral Fun Pack

Evaluate the following line integrals, for fun, and as a **small part** of your preparation for your exam!
Use FTLI or Green's Theorem when you can. Unless otherwise specified, assume a CCW orientation on all closed curves.

1. $\int_C \frac{2x}{x^2+y^2}\,dx + \frac{2y}{x^2+y^2}\,dy$ where C is the part of the clockwise oriented circle $(x-6)^2+(y-3)^2=49$ from (13,3) to (-1,3)
2. $\int_C xy^4\,ds$, where C is the right half of the circle $x^2+y^2=16$
3. $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F}(x,y,z)=(yz,xz,xy)$ and C is given by $\vec{r}(t)=(t,t^2,t^3)$, $0\le t\le 2$
4. $\int_C x^2y\sqrt{z}\,dz$, where C is given by $x=t^3$, $y=t$, $z=t^2$, $0\le t\le 1$
5. $\int_C (2x+9z)\,ds$, where C is given by $x=t$, $y=t^2$, $z=t^3$, $0\le t\le 1$
6. $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F}(x,y)=(x^2y^2,4xy^3)$ and C is the triangle with vertices (0,0), (1,3), and (0,3)
7. $\int_C xe^y\,ds$, where C is the arc of the curve $x=e^y$ from (1,0) to $(e,1)$
8. $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F}(x,y)=(y,x+2y)$ and C is the upper semicircle starting at (0,1) and ending at (2,1)
9. $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F}(x,y)=(xe^{-2x},x^2+2x^2y^2)$ and C is the boundary of the region between the circles $x^2+y^2=1$ (oriented CW) and $x^2+y^2=4$ (oriented CCW).
10. $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F}(x,y,z)=(\sin x,\cos y,xz)$ and C is given by $\vec{r}(t)=(t^3,-t^2,t)$, $0\le t\le 1$
11. $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F}(x,y,z)=(y^2\cos z,2xy\cos z,-xy^2\sin z)$, where C is given by $x=t^2$, $y=\sin t$, $z=t$, $0\le t\le 1$
12. $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F}(x,y)=(x^3y^4,x^4y^3)$, C is $x=\sqrt{t}$, $y=1+t^3$, $0\le t\le 1$
13. $\int_C xy\,dx+(x-y)\,dy$, where C consists of line segments from (0,0) to (2,0) and from (2,0) to (3,2)
14. $\int_C e^y\,dx+2xe^y\,dy$, where C is the square with sides $x=0$, $x=1$, $y=0$, and $y=1$, oriented CW.
15. $\int_C \sin y\,dx+x\cos y\,dy$, where C is the ellipse $x^2+xy+y^2=1$.
16. $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F}(x,y,z)=(2xz+y^2,2xy,x^2+3z^2)$, where C is any curve from (0,1,-1) to (1,2,0)
17. $\int_C x^2\,dx+y^2\,dy+z^2\,dz$, where C consists of line segments from (0,0,0) to (1,2,-1) and from (1,2,-1) to (3,2,0)
18. $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F}(x,y)=(x^2y^3,-y\sqrt{x})$ and C is given by $\vec{r}(t)=(t,-t^3)$, $0\le t\le 1$
19. $\int_C z\,dx+x\,dy+y\,dz$, where C is given by $x=t^3$, $y=t^3$, $z=t^2$, $0\le t\le 1$
20. $\int_C y/x\,ds$, where C is given by $x=t^4$, $y=t^3$, $1/2\le t\le 2$
21. $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F}(x,y,z)=(e^y,xe^y,(z+1)e^z))$, where C is given by $x=t$, $y=t^2$, $z=t^3$, $0\le t\le 1$
22. $\int_C xy^3\,ds$, where C is given by $x=4\sin t$, $y=4\cos t$, $z=3t$, $0\le t\le \pi/2$
23. $\int_C (xy+\ln x)\,dy$, where C is the arc of the parabola $y=x^2$ from (1,1) to (3,9)
24. $\int_C xy\,ds$, where C is the arc of the parabola $y=x^2$ from (1,1) to (3,9)
25. $\int_C y^3\,dx-x^3\,dy$, where C is the circle $x^2+y^2=4$.
26. $\int_C y\,ds$, where C is given by $x=t^2$, $y=t$, $0\le t\le 2$
27. $\int_C x^2z\,ds$, where C is the line segment from (0,6,-1) to (4,1,5)
28. $\int_C xe^y\,dx$, where C is the arc of the curve $x=e^y$ from (1,0) to $(e,1)$
29. $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F}(x,y)=(\frac{y^2}{1+x^2},2y\arctan x)$, C is $x=t^2$, $y=2t$, $0\le t\le 1$
30. $\int_C (x+yz)\,dx+2x\,dy+xyz\,dz$, where C consists of line segments from (1,0,1) to (2,3,1) and from (2,3,1), to (2,5,2)
31. $\int_C (y+e^{\sqrt{x}})\,dx+(2x+\cos y^2)\,dy$, where C is the boundary of the region enclosed by the parabolas $y=x^2$ and $x=y^2$.
32. $\int_C xe^{yz}\,ds$, where C is the line segment from (0,0,0) to (1,2,3)
33. $\int_C ye^x\,ds$, where C is the line segment joining $(1,2)$ to $(4,7)$
34. $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F}(x,y,z)=(z,y,-x)$ and C is given by $\vec{r}(t)=(t,\sin t,\cos t)$, $0\le t\le \pi$

Math 250 Line Integral Fun Pack – Hints and Answers

Please email answers to me as you work through these problems, and I will keep an updated list of answers on our website.

1. FTLI, $\ln 10 - \ln 178$
2. RV, $2^{13}/5$
3. VF, 64
4. VF, 1/5
5. RV, $\frac{1}{6}(14^{3/2} - 1)$
6. Green, 318/5
7. RV, $\frac{1}{3}[(e^2+1)^{3/2} - 2^{3/2}]$
8. FTLI, 2
9. Green, 0
10. VF, $\sin(-1) - \cos 1 + 6/5$
11. FTLI, $\sin^2 1 \cos 1$
12. FTLI, 4
13. VF, 17/3
14. Green, $1 - e$
15. FTLI or Green, 0
16. FTLI, 5
17. VF, 35/3
18. VF, $-\frac{1}{12} - \frac{6}{13}$
19. VF, 3/2
20. RV, $\frac{1}{48}(73^{3/2} - 13^{3/2})$
21. FTLI, $2e$
22. RV, 320
23. VF, $\frac{484}{5} + 9\ln 3 - 4$
24. RV, $\frac{1}{16}[\frac{1}{5}(37^{5/2} - 5^{5/2}) - \frac{1}{3}(37^{3/2} - 5^{3/2})]$
25. Green, -24π
26. RV, $\frac{1}{12}(17^{3/2} - 1)$
27. RV, $\sqrt{77}(56/3)$
28. VF, $(e^3 - 1)/3$
29. FTLI, π
30. VF, 97/3
31. Green, 1/3
32. RV, $\frac{\sqrt{14}}{12}(e^6 - 1)$
33. RV, $2\sqrt{34}e^4$
34. VF, π