Line and Surface Integrals – Summary of Notation

Curves

1. $\vec{r}(t) = (x(t), y(t), z(t))$ is parametrization of curve
2. $d\vec{r} = (dx, dy, dz)$ = path differential
3. $ds = \|d\vec{r}\| = \sqrt{dx^2 + dy^2 + dz^2} = \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\ dt$ = arc length differential
4. Arc length $L = \int_C ds = \int_C \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\, dt$
5. Real-valued line integral: if f is real valued, then path integral is $\int_c f\ ds = \int_a^b f(\vec{r}(t))\, \|\vec{r}'(t)\|\, dt$
6. Vector field l ine integral: if $\vec{F}$ is a vector field, then line integral is $\int_c \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t)\, dt$

Surfaces

1. $\vec{r}(u, v) = (x(u, v), y(u, v), z(u, v))$ is parametrization of surface
2. $d\vec{S} = \vec{r}_u \times \vec{r}_v$ = surface element = normal vec to bit of surface; takes into account surface area
3. $dS = \|d\vec{S}\| = \|\vec{r}_u \times \vec{r}_v\|\, du\, dv$ = surface *area* element
4. Surface area $A = \int\int_D dS = \int\int_D \|\vec{r}_u \times \vec{r}_v\|\, du\, dv$
5. Integrating real valued function over surface: $\int\int_S f dS = \int\int_D f(\vec{r}(u, v)) \|\vec{r}_u \times \vec{r}_v\|\, du\, dv$
6. Surface Integral of vector field: $\int\int_S \vec{F} \cdot d\vec{S} = \int\int_D \vec{F}(\vec{r}(u, v)) \cdot (\vec{r}_u \times \vec{r}_v)\ du\ dv$