Math 250 Surface Integral Fun Pack !

Of course, this is just the beginning. You should do many, many more to prepare for your final!

1. Calculate the outward flux of $\vec{F}(x, y, z) = (x, 2y, 3z)$ across the boundary of the first-octant unit cube with opposite vertices (0,0,0) and (1,1,1).

2. Calculate the line integral of $\vec{F} = (x^2y^3, 1, z)$ over the curve created by the intersection of the cylinder $x^2 + y^2 = 4$ and the top half of $x^2 + y^2 + z^2 = 16$, oriented CW when viewed from above.

3. Evaluate the integral of $\vec{F}(x, y, z) = (x, y, 0)$ over the surface S, which is the hemisphere $z = \sqrt{9 - x^2 - y^2}$ with upward pointing normal vector.

4. Integrate $\vec{F}(x, y, z) = (x, y, z^4)$ over part of the cone $z = \sqrt{x^2 + y^2}$ beneath the plane $z = 1$ with downward orientation.

5. Calculate the inward flux of $\vec{F}(x, y, z) = (0, 0, z^2)$ across S, which is the boundary of the solid bounded by the paraboloids $z = x^2 + y^2$ and $z = 18 - x^2 - y^2$.

6. Calculate the flux of the curl of $\vec{F}(x, y, z) = (3y, 5 - 2x, z^2 - 2)$ over the top half of the sphere of radius $\sqrt{3}$, with upward orientation.

7. Integrate $\vec{F}(x, y, z) = (y, -x, 0)$ over the surface S that is the part of the cone $z = r$ with outward orientation that lies within the cylinder $r = 3$.

8. Evaluate the surface integral of $\vec{F}(x, y, z) = (xy, yz, zx)$ over the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the square $0 \leq x \leq 1$ and $0 \leq y \leq 1$, orientated downward.

9. Integrate $\vec{F}(x, y, z) = (xze^y, -xze^y, z)$ over the part of the plane $x + y + z = 1$ in the first octant with downward orientation.

10. Compute $\oint_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (\sin x^2, e^{y^2} + x^2, z^4 + 2x^2)$ and C is the triangle with edges from (3,0,0) to (0,2,0) to (0,0,1) and back to (3,0,0).

11. Integrate $\vec{F}(x, y, z) = (y, -x, 0)$ over the surface S that consists of the part of the cone $z = r$ that lies within the cylinder $r = 3$, together with the lid made of a disk of radius 3 at height $z = 3$, oriented inward.

12. Find the integral of $\vec{F}(x, y, z) = (x, y, z)$ over S, where S is the part of the plane $z = 3x + 2$ (with upward orientation) that lies within the cylinder $x^2 + y^2 = 4$.

13. Calculate $\iint_S \vec{F} \cdot \ d\vec{S}$, where $\vec{F}(x, y, z) = (x, y, z)$ and S is the sphere $x^2 + y^2 + z^2 = 9$. Use the outward normal.

14. Compute $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x, y, z) = (-y \sin x + \frac{y^3}{3}, \cos x + \frac{x^3}{3}, xyz)$ and C is the circle $x^2 + y^2 = 1$ in the plane $z = 1$, oriented CCW when viewed from above.

15. Evaluate the integral of $\vec{F}(x, y, z) = (0, y, -z)$ over S, which consists of the paraboloid $y = x^2 + z^2$, $0 \leq y \leq 1$, and the disk $x^2 + z^2 \leq 1, y = 1$. Use the outward normal.

16. Integrate the curl of $\vec{F}(x, y, z) = (x, y, z^4)$ over part of the cone $z = \sqrt{x^2 + y^2}$ beneath the plane $z = 1$ with downward orientation.

17. Calculate the flux of $\vec{F}(x, y, z) = (x, -y, 0)$ outward across the boundary of the solid first-octant pyramid bounded by the coordinate planes and the plane $3x + 4y + z = 12$.

Math 250 Surface Integral Fun Pack Hints and Answers !

1. 6, divergence
2. 8π, stokes
3. 36π, directly
4. $\pi/3$, directly
5. -1458π, divergence
6. -15π, stokes
7. 0, directly
8. $-713/180$, directly
9. $-1/6$, directly
10. 0, stokes
11. 0, divergence
12. 8π, directly
13. 108π, divergence
14. 0, stokes
15. 0, divergence
16. 0, stokes
17. 0, divergence