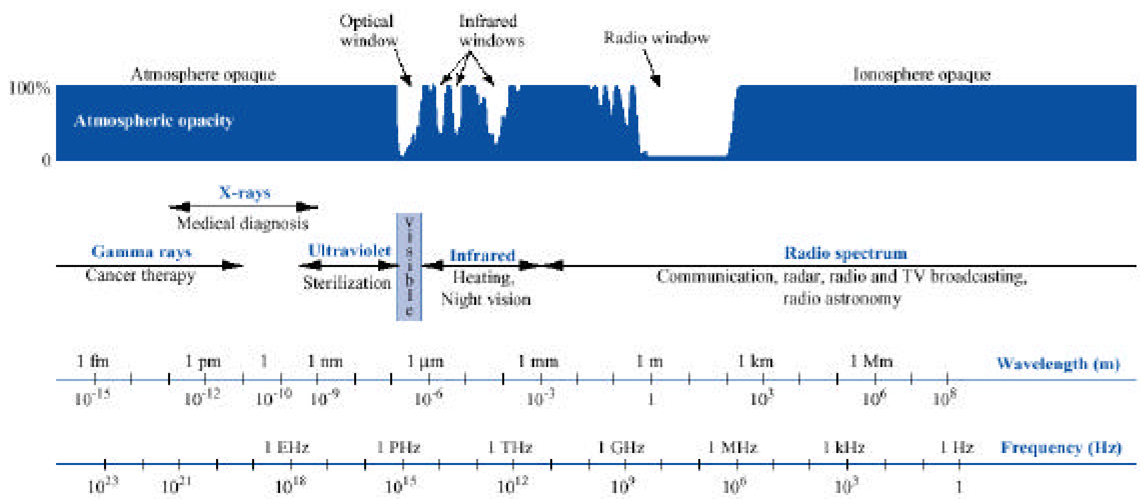
Lecture 1: Importance of Electromagnetics

Electricity and magnetism in three dimensions is understood through electromagnetic theory. Even simple circuit theory is founded on electromagnetic principles.

DIMENSIONS AND UNITS

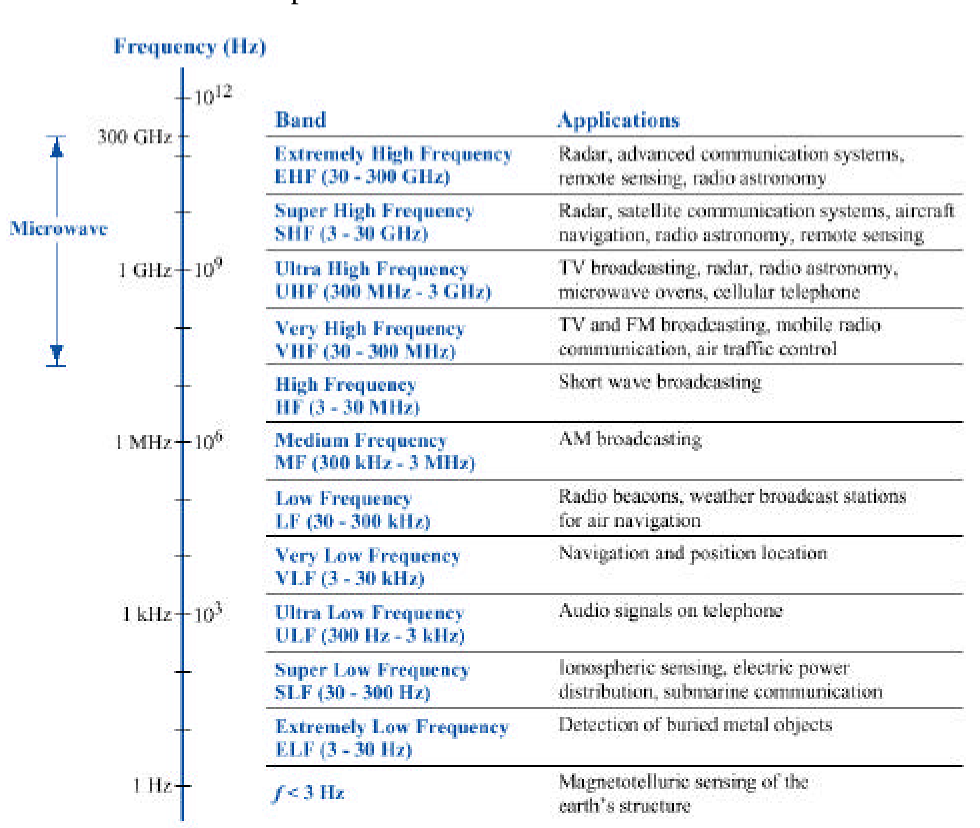
We will use *Système Internationale d'Unités* or the SI system of units whose fundamental units are: kilogram, meter, second, Ampere, Kelvin, and Candela for mass, length, time, temperature, and luminous intensity, respectively.

We will also work with frequency and wavelength of electromagnetic fields. The electromagnetic spectrum is shown:



The Electromagnetic Spectrum

Of some interest to us is the radio spectrum shown below:



The Radio Frequency Band

Frequencies of 1 GHz and above are defined in specific Microwave Bands:

New Band Designation (old designation)	Frequency
D (L)	1 - 2 GHz
E, F (S)	2 - 4 GHz
G, H (C)	4 - 8 GHZ
I, J (X)	8 - 12 GHz
J (Ku)	12 - 18 GHz
J (K)	18 - 26 GHz
K (Ka)	26 - 40 GHz

SYMBOLS USED

Quantities or dimensions like charge Q or mass M are in italics. Vectors are bold as in electric field vector **E**. The magnitude of **E** is a scalar quantity E. Unit vectors are bold with a hat over the letter: for example, $\hat{\mathbf{x}}$. A sample notation is:

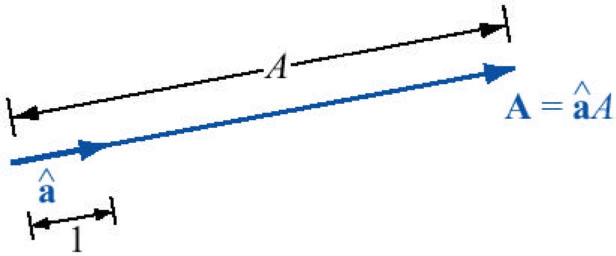
$$\mathbf{F} = \hat{\mathbf{x}}\ 200\ \text{kg-m-s}^{-2}$$

Please also remember to use dimensional analysis.

VECTOR ANALYSIS

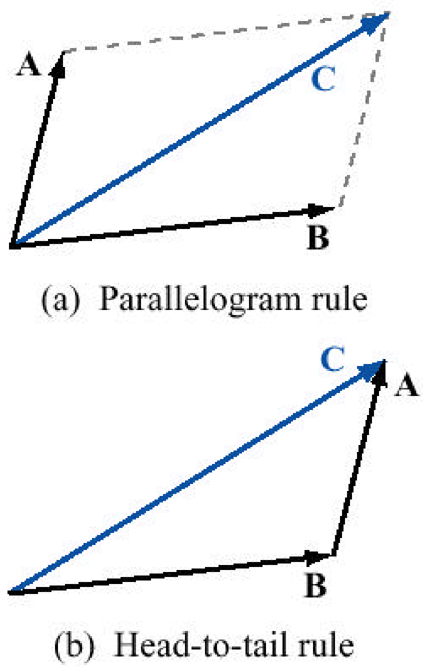
A *scalar* is a quantity that has magnitude only.

A *vector* has both magnitude and direction.



Vector **A** = **â** A has magnitude A = | **A** | and unit vector **â** = **A**/*A*

VECTOR ADDITION

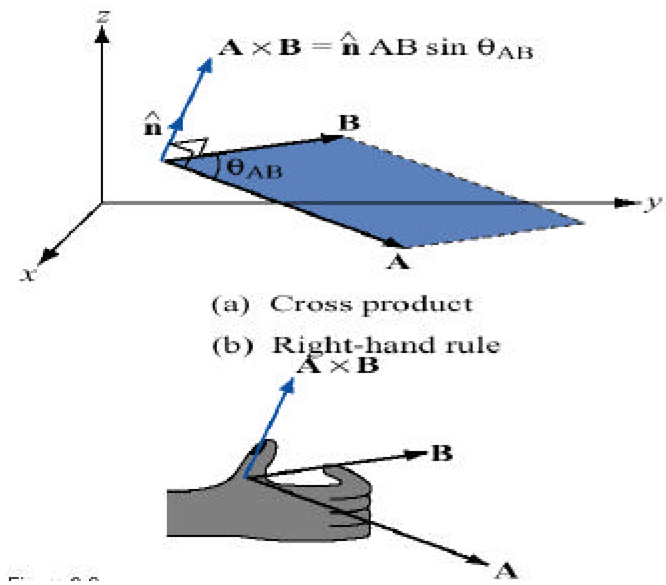


Vector addition using the parallelogram rule and the head-to-tail rule

In the figure above for vector addition: **C** = **A** + **B**.

RECTANGULAR COORDINATES AND VECTOR COMPONENTS

A rectangular or cartesian coordinate system has three mutually perpendicular axes called the x, y and z axes as shown below. We will use the right-handed system.



Cartesian Coordinate System

Any vector can be resolved into three components each parallel to the coordinate axes:

$$\mathbf{A} = \mathbf{A}_\mathbf{x} + \mathbf{A}_\mathbf{y} + \mathbf{A}_\mathbf{z} \;.$$

Althernately, the vector **A** can be represented in terms of unit vectors:

$$\mathrm{A} = \hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z \;.$$

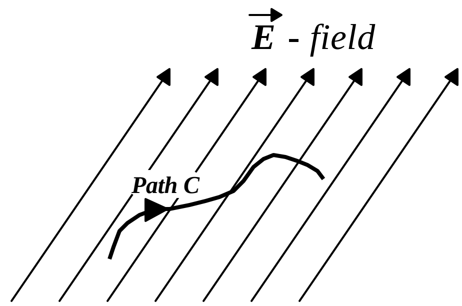
The magnitude of the vector A is $|\mathbf{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$.

The dot product is defined as: $\mathbf{A}\bullet\mathbf{B} = AB\cos ?_{AB}$.

LINE INTEGRAL

The line integral is the integral of the dot product of a vector field with a specified path, *C*, shown in the figure below. In practice, the integral is taken over the length of the path of the dot product of the vector field and incremental vectors along the specified path. The form of the line integral is:

$$V = \int_C \mathbf{E} \bullet d\mathbf{L}$$



Representation Of Field And Path For Line Integral

For instance to determine the incremental work $d\mathbf{W}$ doen by a force **F** in moving an object a distance cosθ $dL = dr$ is

$$d\mathbf{W} = \mathbf{F} \bullet d\mathbf{L} = F \cos\theta \, dL \ .$$

The line integral is then the total work W done by **F** on an object moved over a path from P_1 to P_2 :

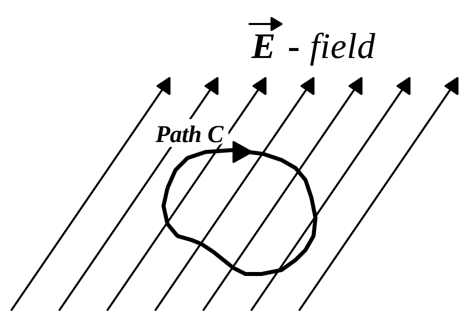
$$W = \int_{P_1(start)}^{P_2(end)} \mathbf{F} \bullet d\mathbf{L}$$

LINE INTEGRAL OF A CLOSED PATH

When the path under consideration is a closed path, C, as shown below, such that the path has no beginning and no ending like a rubber band, the line integral is written with a circle associated with the integral sign:

$$\oint_C \mathbf{E} \bullet d\mathbf{L} = \iint_S (\nabla \times \mathbf{E}) \bullet d\mathbf{S}$$

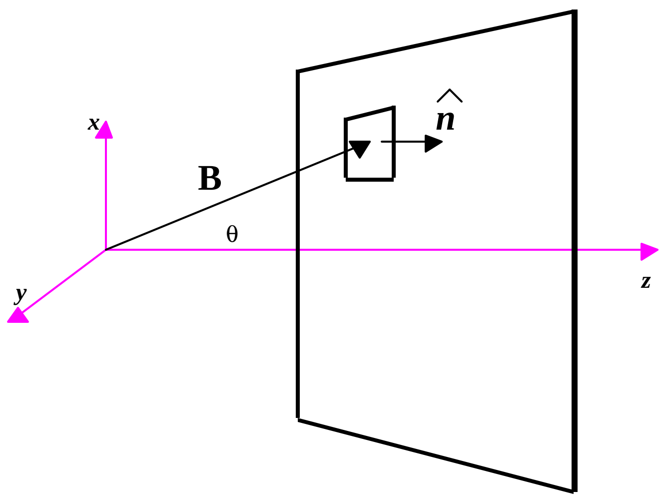
The line integral of a vector field around a closed path is known as the circulation of that vector field. In particular, the line integral of **E** around a closed path is the work per unit charge done by the field in moving a test charge around the closed path.



Representation Of Field And Closed Path For Line Integral

SURFACE INTEGRAL

Surface integration is merely the adding up of the normal components of a vector field over a given surface *S*, as shown in the figure below. Assume that the surface is broken up into incremental surface areas, each with a normal component *d***S**. The vector *d***S** is normal to its incremental surface. For a closed surface, *d***S** is directed outward by convention. Otherwise, choose between the two normal directions using physical reasoning. Use the vector field **V** at the position of the incremental surface element, and take its dot product with *d***S**: the result is a differential (small) scalar. The sum of these scalar contributions over all the incremental surface elements us the surface integral: which is also a scalar.



Vectors Used To Find Surface Integral

If we define an area as a vector of magnitude A (scalar area of the loop area) and direction perpendicular to its surface as shown above, we can express the rate of flow or flux of as:

$$y = BA\cos q = \mathbf{B} \bullet \mathbf{A}$$

The we integrate the contributions at all points across the surface of the loop (surface integral) obtaining total flux:

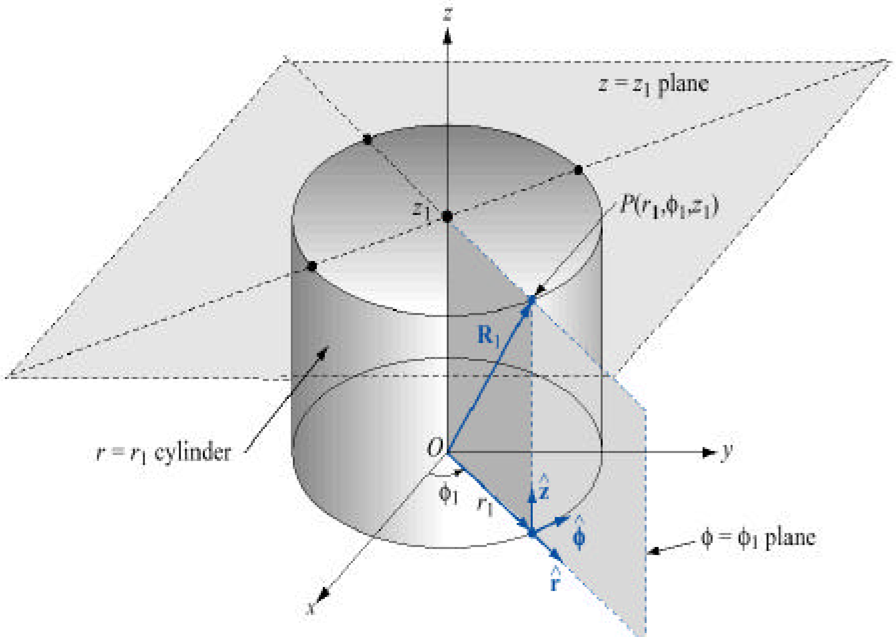
$$y = \iint_{\substack{Area\\A}} \mathbf{B}\bullet\hat{\mathbf{n}}\,ds = \iint_{\substack{Area\\A}} \mathbf{B}\bullet d\mathbf{s}$$

CROSS PRODUCT

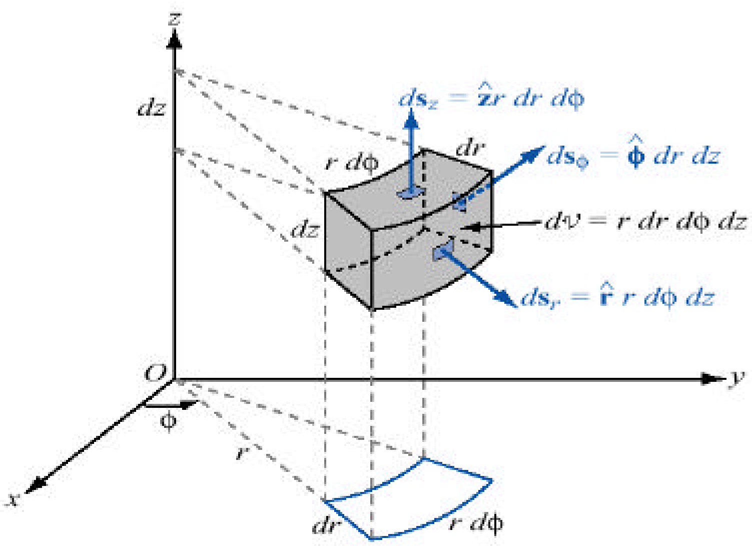
The cross product is defined as:

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{\mathbf{x}}\left(A_yB_z - B_yA_z\right) - \hat{\mathbf{y}}\left(A_xB_z - B_xA_z\right) + \hat{\mathbf{z}}\left(A_xB_y - B_xA_y\right)$$

CYLINDRICAL COORDINATE SYSTEM

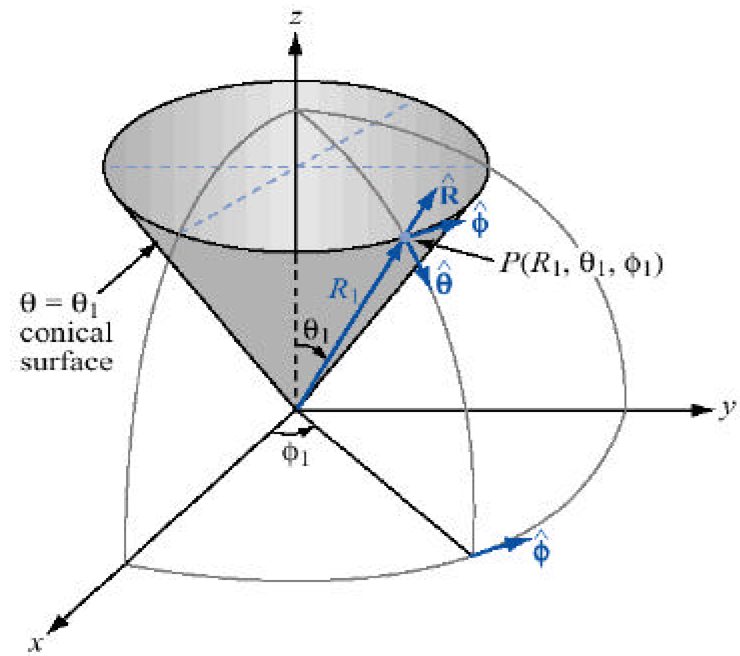


The Spherical Coordinate System

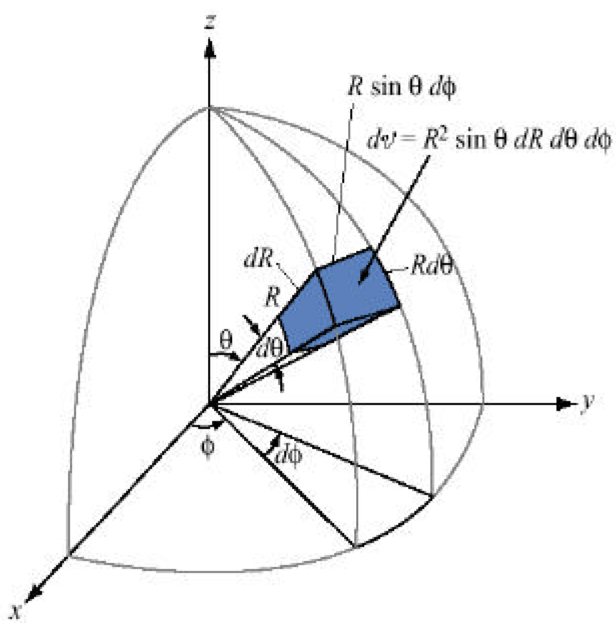


Differential Areas and Volume in cylindrical coordinates

SPHERICAL COORDINATE SYSTEM



Point P (R_1, q_1, f_1) in Spherical Coordinates



Differential Volume in Spherical Coordinates

COORDINATE TRANSFORMATIONS:

cart ↔ cyl

$$\begin{cases} x = r\cos\theta, \\ y = r\sin\theta, \end{cases} \qquad \begin{cases} r = \sqrt{x^2+y^2}, \\ \theta = \arctan\dfrac{y}{x}, \end{cases} \qquad \begin{cases} \sin\theta = \dfrac{y}{\sqrt{x^2+y^2}}, \\ \cos\theta = \dfrac{x}{\sqrt{x^2+y^2}}. \end{cases}$$

cart ↔ sph

$$\begin{cases} x = r\cos\theta\sin\phi, \\ y = r\sin\theta\sin\phi, \\ z = r\cos\phi, \end{cases} \qquad \begin{cases} r = \sqrt{x^2+y^2+z^2}, \\ \theta = \arctan\dfrac{y}{x}, \\ \phi = \arctan\dfrac{\sqrt{x^2+y^2}}{z} \\ \quad = \arccos\dfrac{z}{\sqrt{x^2+y^2+z^2}}. \end{cases}$$

For detailed tables of coordinate parameters and definitions, pleaser refere to Tables 1-4, 1-5, 1-6, and 1-7.

<u>REFERENCES</u>

J. D. Kraus and D. A. Fleisch, Electromagnetics with Applications, Fifth Editon, WCB/McGraw-Hill, 1999.

F. T Ulaby, Fundamentals of Applied Electromagnetics, Prentice-Hall, 1999.